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Peer produced peer learning :

A mathematics case study

by

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A thesis submitted in partial fulfillment of the requirements
for the degree of Doctor of Philosophy.

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The Open University, Milton Keynes, UK
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Abstract

This research project develops around a technological intervention intended to transform a peer produced reference resource into a peer produced learning environment. Through the work described in this thesis, PlanetMath.org, an early online community devoted to mathematics, has now become a mathematical practicum, and a laboratory for learning science.

A new theory that describes the nexus of peer production and peer learning is foundational for the research programme. The candidate theory was initially developed during a pilot study based on online field work at the Peer-2-Peer University. The new theory – which is given the name "paragogy" – has implications for designers, researchers, educators, and others whose work relies on peer learning and peer production. Further research and development work in the PlanetMath context helped to refine the theory, and applied it along with a range of mixed methods to develop an anthropologically-inspired study of modern mathematics.

A quantitative approach was employed to detect the factors of interaction that influence learning outcomes, using legacy data from PlanetMath. A qualitative, interview-based approach was employed, to understand the desiderata of potential users of a new system emphasizing peer learning. The new software system was implemented, informed by paragogy and these stakeholder perspectives, using Drupal and other open source components. Field work with PlanetMath users after the launch of the new system employed an emergent design process to elaborate the theory and develop a viable approach to ongoing development and codesign.

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Peter Scott and Alexander Mikroyannidis supervised my work. Marian Petre and Martin Oliver examined it. I think they will all agree it has been a lot of work to get to this point.

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> *There is a pool of blood somewhere, a place you came from. You will find this blood petrified into stone and it is red. It comes from a sacred spot common to all people, where even enemies are turned into friends and relatives.*
>
> *Lame Deer, Seeker of visions*, 1972

Among all my friends and relations I particularly wish to thank my father, Steve Corneli. I remember when I asked him for help with a geometry problem, and he said to me: "Do you just want the answer, or do you want to really understand how it works?" I was far too ashamed to admit that what I wanted was the answer, so I think I said, "Both." I thought about it some more afterwards, and realized, what I really wanted was a computer program that would help me understand how to get the answer. Well, I don't have the computer program yet. But I know a lot more about how it works.

Figure 4 is © 2013 Gerry Stahl and is used here with permission.

Figure 5 is © 2013 Sean Goggins and is used here with permission.

Tables 11, 12, and parts of Chapter 9 appear in altered form in the *Peeragogy Handbook*.

Appendix B draws extensively on notes from a public lecture on "Modelling the mathematical discovery process" presented by Sir Timothy Gowers in his Maxwell Institute Lecture at the University of Edinburgh, November 2, 2012.

Publications

Eleven recent peer reviewed publications helped to explicate the main ideas in the thesis in the context of academic dialog. These papers were published in venues devoted to wikis, open knowledge, learning design, learning science, computer mathematics, and commons governance. The thesis draws on and complements recent experiments with research-through-writing (Tomlinson et al., 2012), and joint work on the *Peeragogy Handbook* (Rheingold et al., 2014). The benefits of collaboration in research are particularly clear for research on collaboration, as this allows theoretical ideas about the topic of study to be developed and grounded in practice. The associated publication track record reflects the interdisciplinary and mediatedly-social nature of this project. That said, in conformity with academic norms, the text presented here remains the responsibility of a single author.

Chapter 2 (Historical Background):

- Joseph Corneli (2011), The PlanetMath Encyclopedia, in *Workshop on Mathematical Wikis (MathWikis-2011) at ITP (2nd International Conference on Interactive Theorem Proving) 2011, Nijmegen, NL, August 27th, 2011.*

  This paper provides a retrospective look at PlanetMath's first decade. This material is used to to develop a dispassionate perspective on PlanetMath's strengths and weaknesses.

Chapter 3 (Theoretical Background):

- Joseph Corneli and Alexander Mikroyannidis (2012), Crowdsourcing Education on the Web: A Role-based Analysis of Online Learning Communities, in Alexandra Okada, Teresa Conolly, and Peter Scott (eds.), *Collaborative Learning 2.0: Open Educational Resources*, IGI Global.

  This book chapter provides an analysis of learning in terms of its constituent roles. This is reassessed here, to better emphasize the importance of context.

- Joseph Corneli and Charles Jeffrey Danoff (2011), Paragogy, in *OKCon 2011: Proceedings of the 6th Open Knowledge Conference.*

  This was the first published paper on the paragogy framework, developed in a pilot study at the Peer-2-Peer University. Paragogy is the central idea that is developed in the thesis.

– Joseph Corneli and Alexander Mikroyannidis (2011). Personalised and Peer-Supported Learning: The Peer-to-Peer Learning Environment (P2PLE), *Digital Education Review*, 20.

  This paper reinterprets paragogy as a set of design principles. These are drawn upon to present a understand stakeholder perspectives on the PlanetMath re-build.

– Joseph Corneli, Paragogical Praxis, *E-Learning and Digital Media*, 9(3), 2012.

  This paper looks at the methods that people can use to "do" paragogy. Together with the design principles mentioned above, this work is used to develop a consistent framework for analysis of peer produced peer learning in terms of system change.

Chapter 5 (Methodology):

– Joseph Corneli and Marisa Ponti (2012), Detecting mathematics learning online, in Hodgson V., Jones C., de Laat M., McConnell D., Ryberg T. & Sloep P. (eds.), *Proceedings of the 8th International Conference on Networked Learning 2012*, 2-4 April, 2012, Maastrict, NL.

  This paper considers the prospects for modeling the causes of learning in an online context. The paper presented foundational designs for both a quantitative analytic approach, and a qualitative heuristic approach. These ideas are sharpened in Chapter 5 to describe the reseach methods that will be used in subsequent chapters.

Chapter 8 (Implementation):

– Joseph Corneli, Constantin Jucovschi, and Alexander Mikroyannidis (2011). PlanetMath Redux: Web 2.0 infrastructure for mathematical problem solving, in *Technology-Enhanced Learning for Mathematics and Science: Landmark Research and New Contributions (TELMAS@ECTEL'2011), September 21, 2011, Palermo, Italy.*

  This paper presented a preliminary outline of what it would mean to convert PlanetMath into a "learning environment." This is expanded here with a requirements analysis informed by research with stakeholders.

– Joseph Corneli and Mircea Alexandru Dumitru (2012). PlanetMath/Planetary, in *Proceedings of Works in Progress track, MathUI and OpenMath workshops at the Conferences on Intelligent Computer Mathematics 2012.*

  This paper presented a technical overview of the Drupal 7 based rebuild of PlanetMath's infrastructure. Chapter 8 presents an analysis of the deployed system, including screenshots.

Chapter 9 (User evaluation):

– Joseph Corneli, Dorota Marciniak, Charles Jeffrey Danoff, Charlotte Pierce, Paola Ricaurte, Jan Herder, Suz Burroughs, George Brett and John Graves (2014). Building the Peeragogy Accelerator, in Megan Quentin-Baxter, ed., *Proceedings of OER14: building communities of open practice*,

  This paper describes work-in-progress on project-to-project collaboration in the Peeragogy project, supported by a design pattern methodology. The approach to collaboration described in this paper informed my interpretation of user feedback on PlanetMath/Planetary.

Chapter 10 (Discussion):

– Joseph Corneli, Anna Keune, Amanda Lyons, and Charles Jeffrey Danoff (2013). Peeragogy in Action, in Kaitlyn Braybrooke, Jussi Nissilä and Timo Vuorikivi, eds., *The Open Book*, #3 in the Reaktio Series, published by The Finnish Institute in London, UK. ISBN 978-0-9570776-3-8.

  One key question in my discussion chapter is whether PlanetMath is useful for instructors, in practice. This book chapter a presents a general-purpose design for peer learning activities; it is instantiated here with a concrete example.

– Joseph Corneli (2014), Towards a Creativity Commons, *Fifth "Workshop on the Ostrom Workshop" (WOW5)*. Working group: "The Commons as Transformative Paradigm".

  This paper helped to clarify the philosophical issues underlying the design pattern methodology. I draw on this work to help explain the broader implications of the research presented here.

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Part 1
Introduction

1 Studying Mathematics the Commons-Based Peer Production Way

Mathematics is a socially shared, symbolically – and technologically – mediated *cultural* activity. This suggests that one way to understand mathematics better is to look at how people *do* mathematics.

1.1 What this is and is not

This does not mean watching over individuals who are solving mathematics problems and taking notes on how often they look out the window and whether or not they ultimately get the correct answer. Nor will this study rely, for the most part, on the even more typical (and obvious) idea: being that person oneself. As Eugene Matusov argues:

> *Focusing on the individual as the unit of analysis blinds researchers and educators from seeing the systemic complexity of psychological phenomena and from considering systemic solutions to complex psychological and social problems.* (Matusov, 2007)

Rather than studying mathematics in the traditional way, by solving mathematics problems – or focusing exclusively on studying discrete instances of others' problem solving – the proposal is, to the extent possible, to study mathematics as a cultural system. In itself this perspective is not new. Ludwig Wittgenstein wrote that "*mathematics is after all an anthropological phenomenon*" (Wittgenstein, 1956, §V.26). Wittgenstein looked at the foundations of mathematics primarily through the lens of language. For example: "*A definition signifies a change in technique*" (Wittgenstein, 1976, p. 33).

In 1997–1998, as a beginning student of cultural anthropology and multivariable calculus, it occurred to me that mathematics was perhaps the ideal example of "culture",

when this is understood to mean:

> [S]*ocially acquired, symbolically mediated systems of techno-ecological adaptive strategy, organization* [...] *and cosmology*[.]
>
> Luther P. Gerlach, quoted in (Brandt, 2005)

At the time, my ambition to study mathematics as a participant observer meant doing the obvious thing and becoming a mathematics student. It was only later, when I became acquainted with current computational and computer-aided approaches to mathematics, that the foundations for the current study would begin to develop in earnest.

What I have more recently observed and participated actively in, and what will be described here, is the collaborative design and development of an online learning space for mathematics. This work features mathematical culture at its own most "cultural" level, and evokes "soft" cultural patterns – for instance, values pertaining to enculturation. To talk about enculturation rather than learning here is "*emphasizing not just a conditioning process but an adaptation process*" (Brandt et al., 2006). Indeed, at least in the current setting, this is a dual adaptation: not only of people to culture, but of the culture itself.

This study will explore the ways people think about mathematics and ramify their "native" theories and practices of mathematical culture. It will develop a theory of *peer produced peer learning*, inspired by and as a reflection on mathematical practice, but with broader implications. The work draws inspiration from anthropology and philosophy, but the methods and primary outcomes pertain to humanistic computing and to computer science. The following sections describe the research setting and approach, indicate the main scientific and practical contributions, and outline the argument of the thesis.

1.2 Setting and audience

> *PlanetMath is a virtual community which aims to help make mathematical knowledge more accessible. PlanetMath's content is created collaboratively: the main feature is the mathematics encyclopedia with entries written and reviewed by members.*
>
> Aaron Krowne, c. 2001, on HTTP://PLANETMATH.ORG

The PlanetMath encyclopedia currently defines over 15000 mathematical terms, covering both basic and advanced mathematical topics. Each page in the encyclopedia is accompanied by a threaded forum discussion. The development of the site and its highly custom software was the central topic in Aaron Krowne's Masters thesis, *An Architecture for Collaborative Math and Science Digital Libraries* (Krowne, 2003A).

PlanetMath's byline is "*math for the people, by the people.*" It is an example of a resource created through "*non proprietary production by peers who do not interact either through a firm or through a market*" (Benkler, 2002); in short, PlanetMath is *peer produced.* As Chapter 2 explains, a particular sort of wholly virtual ownership underlies PlanetMath's original design, but the site is nevertheless a "*community-managed commons*" (Gardner & Stern, 1996).

Although the encyclopedia is historically PlanetMath's central feature, it is not the project's sole commitment. PlanetMath is better described as a mathematics digital library built "*the commons-based peer production way*" (Krowne, 2003в). In a joint paper exploring potential new directions for the project, we wrote:

> *We find it compelling that, in the context of a digital library, marginal conversations within a text provide a chance for readers to interact with primary authors and with each other, and to become primary authors themselves, all at once. While marginalia are considered to be vandalism in physical library books, in a digital library, there is no reason to fear them – they can easily be hidden away.* (Corneli & Krowne, 2005)

The research question that I will be concerned with here is:

> What is needed to transform this peer produced digital library and mathematics reference resource into a peer produced peer learning environment?

The historical audience of the PlanetMath website was anyone who might consult an online encyclopedic dictionary of mathematics, or who would use a forum to discuss questions about mathematics. This population includes advanced undergraduates, graduate students, mathematics professors, and professionals. The historical co-creators of PlanetMath formed a very particular subset of this population. These were people who were interested in discussing mathematics over the internet, asking questions and sharing existing knowledge, and often learning in the process. Certainly, this was peer produced peer learning, but learning was not the focus – creating an encyclopedia was. Chapter 2 traces the history of the PlanetMath project, and sets the stage for its potential evolution.

My hope is that in the future, PlanetMath's audience – or better still, its public – will expand to include anyone who might consult a mathematics textbook or Schuams outline, or who might take or design an online mathematics course, tutorial, or seminar. Users will be able to collectively inquire into the techniques and approaches that work best, using PlanetMath as a laboratory. Much as GNU/Linux offers a complete free operating system for computers, PlanetMath or its eventual successor may expand to be a go-to place for mathematicians, mathematics teachers, and students at all levels.

For now, however, there are a range of technical issues at stake which limit the scope of the current work to a far more modest scale, and which correspondingly narrows the range of users served. The technical focus of the current work is on building infrastructure that supports sharing, discussing, and solving textbook-style problems at the university or beginning graduate level. A prototypical user would be anyone doing independent study of mathematics at this level – whether or not they are formally enrolled as a student. The particular audience that the research focuses on are those socially-minded independent learners who would be interested in benefiting from or contributing to peer support using this technology.

A concrete use case can be described, building on an essay by computer scientist Clayton Lewis, who writes about the new potential of "DIY Education". In order to explore this theme, Lewis imagines himself caught in a time warp. Whereas Clayton_{1963} "*lived in the dormitories of the college he attended*," due to the rising costs of education, Clayton_{2010} "*has no such easy choice*" (Lewis, 2010). He looks around for free or low-cost options, and discovers the Peer 2 Peer University.*

> *Following his computer interests, Clayton explores the course offerings at P2P U. He's attracted by the Python programming course, and also by a cooperative program in Web technology offered jointly by P2P U and the Mozilla Foundation, World of Webcraft. He also thinks about Differential Equations for Engineers, but is put off by the fact that it isn't pure math, something about which he has a childish prejudice. He does sign up for DIY Math, facilitated by a doctoral student at the Open University, because it is open as to topic. He has a vague idea that there should be some interesting mathematics around games, not what is called "game theory", which he has read enough about to know isn't what he's interested in, but something that allows one to analyze games like penning the pig. (He doesn't know that this is called "combinatorial game theory".) He's hoping that the people in DIY Math can help him find something like that. P2P U has no requirements structure, so he doesn't sign up for anything outside his areas of interest.* (Lewis, 2010)

However, as Chapter 3 will demonstrate, *DIY Math* would actually be a singularly bad choice. One problem is that in following his own interests, Clayton_{2010} "*doesn't have the guiding hand of curriculum* [...] *because P2P U doesn't have a curriculum.*" Nevertheless, Lewis points out that:

> *Anyone could create* [...] *a roadmap for learning in a particular field, or a roadmap for becoming "an educated person" in any more general sense they fancy, and share this via P2P U.* (Lewis, 2010)

One of the key aims behind the changes to PlanetMath is to support the creation, sharing, and critique of such learning roadmaps for the mathematics domain. In addition to peer learning, there is a complementary use case related to peer teaching.

*HTTP://P2PU.ORG

Folded inside of this point there is an important question. If both peer teaching and peer learning are important here, what about other roles? In particular, what about the person or persons who will be involved with creating the website, which offers suitable conditions for others to teach and learn? This points to a very important secondary audience, both for this thesis and the accompanying new software system, which has been been developed and released as free software. These aim to reach the people whose contributions will continue to drive the system forward.

1.3 The relevance of an anthropologically informed approach

The studies developed in this thesis are not ethnographic, but I view them as anthropological in spirit. As Tim Ingold explains, "*anthropology is not ethnography*" (Ingold, 2008). He offers this perspective on what anthropology can be: "*Quite simply, it would be an inquiry into the conditions and possibilities of social life, at all times and everywhere*" – in contrast to the more limited "*objective of ethnography*," which is "*to describe the lives of people other than ourselves.*" A lot hinges on Ingold's understanding of "*social life.*" He writes: "*Societies are not entities analogous to organisms, let alone to machines.*" Indeed, he takes an opposite view: "*Organic life is social.*" Ingold's socially-inspired and process-oriented ontology refers to the "*essential interpenetrability or commingling of mind and world*" and to "*the unfolding of a continuous and ever-evolving field of relations within which beings of all kinds are generated and held in place.*"

Ingold's philosophical views are connected to an approach to inquiry that has significant bearing on the approach that will be taken here.

> *What truly distinguishes anthropology, I believe, is that it is not a study **of** at all, but a study **with**. Anthropologists work and study **with** people. Immersed with them in an environment of joint activity, they learn to see things (or hear them, or touch them) in the ways their teachers and companions do.* (Ingold, 2008), original emphasis

A system like PlanetMath is an approximate description, in an active rather than static form, of the way mathematics is done. At its best, such a system simultaneously creates new ways to do mathematics. In the last analysis, the interpretation of mathematical practice at PlanetMath is an interpretation made together with the user community. Notably, if the system is entirely incongruous with what mathematical practice *is*, then people who are interested in mathematics will have very little use for it!

In expanding the theme of "*studying with*," Ingold emphasizes that he is ultimately far less interested in "*description*" than in "*a practice of correspondence.*" In a recent book bringing anthropology together with archaeology, art, and architecture, he writes:

> *To practise this method is not to describe the world, or to represent it, but to open up our perception to what is going on there so that we, in turn, can respond to it.*
> (Ingold, 2013, p. 7)

For Ingold, after Lave (1990), learning is "*understanding in practice*," and after Gibson (1979), it is based upon "*an education of attention*," which is, drawing on his own fieldwork experiences, "*a process of active following*" (Ingold, 2013, pp. 9, 13, 2, 1). These points are are not discipline-specific: they apply equally to hunting and pottery-making, computer programming and mathematics.

The key feature of Ingold's anthropology that brings it closer to art than to ethnography is that it aims to be "*transformational*" rather than "*documentary*" – here noting that "*no genuine transformation in ways of thinking and feeling is possible that is not grounded in close and attentive observation*" (Ingold, 2013, pp. 3-4).

The fundamental place that Ingold reserves for the social in his thinking bears a similarity to George Herbert Mead's conception, which again emphasizes transformation. For Mead (1932), sociality is the "*stage betwixt and between the old system and the new one*," which occurs whenever a new form of organization is being established, but when neither the new form nor the broader environment has stabilized; *cf.* (Aboulafia, 2012). A view in which the environment is not taken as fixed, but as co-emergent with the people who dwell in it is reflected in the PlanetMath context, in the sense that both content and software are expected to grow and develop along with use.

One further quote can be brought forward to illustrate the essential idea of "*studying with people*" that underlies Ingold's participatory anthropology. This quote describes peer learning in negative form – in other words, it says how things are usually done in mainstream education.

> *Students are told that anthropology is what we do with our colleagues, and with other people in other places, but not with them. Locked out of the power-house of anthropological knowledge construction, all they can do is peer through the windows that our texts and teachings offer them.* (Ingold, 2008)

In the parallel setting of mathematics, there are some exemplary exceptions to this norm. Educator Alan Schoenfeld, who describes *metacognition* as the key to working effectively in mathematics (Schoenfeld, 1987), builds his pedagogical practice around asking his students reflective questions like these:

> *What (exactly) are you doing? Can you describe it precisely? Why are you doing it? How does it fit into the solution? How does it help you? What will you do with the outcome when you obtain it?* (Schoenfeld, 1987, p. 206)

He argues that

> [T]*he interactions among mathematicians, and the sense of community they support, are part of what sustains mathematics – that the practice of mathematics is a human endeavor and very much a cultural one.* [...] [E]*ntry into that culture (or some culture that supports the same values) may be necessary to understand and appreciate mathematics.* (Schoenfeld, 1987, p. 214)

Schoenfeld asserts that, for practicing mathematicians, the classroom "*is an essentially alien culture*," and that "[w]*hat we need is a program of 'cultural design' for schooling.*"

1.4 What (exactly) is to be done

Having described a basic outlook in which mathematics is seen as a cultural and indeed as an already-anthropological affair, it remains to say what concept of culture is most relevant to the study of mathematics, and what will be done here using this concept. The "*socially acquired, symbolically mediated systems*" definition from Gerlach, quoted above, provides a useful big picture, but in its breadth, it does not exactly give a notion of culture that we can easily grasp. It is, however, compatible with a pointed cybernetic notion of culture, developed by Clifford Geertz. First, a bracketing admonition:

> *Culture is not a power, something to which social events, behaviors, institutions, or processes can be causally attributed; it is a context, something within which they can be intelligibly – that is, thickly – described.* (Geertz, 1973, p. 14)

Without yet worrying about the issues attached to these ideas of context and thick description, the following quote provides a working definition of what *culture* itself can be taken to mean:

> *I want to propose two ideas. The first of these is that culture is best seen not as complexes of concrete behavior patterns – customs, usages, traditions, habit clusters – as has, by and large, been the case up to now, but as a set of control mechanisms – plans, recipes, rules, instructions (what computer engineers call "programs") – for the governing of behavior. The second idea is that man is precisely the animal most desperately dependent upon such extragenic, outside-the-skin control mechanisms, such cultural programs, for ordering his behavior.* (Geertz, 1973, p. 44)

So, for Geertz, culture is already-programming![*] Of course it has, by and large, up to now, been programming for humans, not for computers. It is also important to point out that Geertz is explicitly referring to *cybernetic* programming. In other words, his

[*]Geertz's notion of culture derives from that of Talcott Parsons – "*patterned or ordered systems of symbols which are objects of the orientation of action, internalized components of the personalities of individual actors and institutionalized patterns of social systems*" (Parsons, 1951) – "*suitably emended*"; *cf.* (Geertz, 1973, pp. 254), (Peacock, 1981).

concept of cultural control systems "*presupposes that control is partial and not complete*" (Burroughs, 1978).

In light of Ingold's emphasis on trained attention and in-the-world experience, it is not surprising that he is suspicious of the idea of culture as an external control system. In a survey that treats Geertz and other prominent anthropological figures, he writes:

> [O]*ne learns to perceive in the manner appropriate to a culture, not by acquiring programmes or conceptual schemata for organising sensory data into higher-order representations, but by 'hands-on' tasks whose successful fulfilment requires a practised ability to notice and to respond fluently to salient aspects of the environment.*
>
> (Ingold, 2000, p. 167)

And yet, cybernetic control systems are not acquired in the way we might learn a second (or first) language; culture is primarily context, not content; *cf.* (Ingold, 2000, pp. 163–166). Rather, one might say that these systems are present, as more or less salient features of an environment, on the understanding that these features are emergent along with those who dwell in the environment. Whereas Geertz calls "*any object, act, event, quality, or relation which serves as a vehicle for conception*" a "*symbol*" (Geertz, 1973, p. 91), Ingold seems to be more cautious: "*every feature* [of the landscape] *is a potential clue, a key to meaning rather than a vehicle for carrying it*" (Ingold, 2000, p. 208).

Independent of the precise weight we assign to symbol systems as opposed to meaning making, it would be hard to deny that one the greatest cultural shifts in history is connected with the fact that, in recent decades, certain classes of symbolically-mediated programs can be processed with computers. Granted, people continue to perform "*actions that are far beyond the capabilities of any machine yet devised*" (Ingold, 2000, p. 171). What's of interest here is what happens when humans and computers work together.

> *We have been programming universal computers for about 50 years. Programming provides us with new tools to express ourselves. We now have intellectual tools to express "how to" as well as "what is." This is a profound transformation: it is a revolution in the way we think and in the way we express what we think. For example, one often hears a student or teacher complain that the student knows the "theory" of some subject but cannot effectively solve problems. We should not be surprised: the student has no formal way to learn technique. We expect the student to learn to solve problems by an inefficient process: the student watches the teacher solve a few problems, hoping to abstract the general procedures from the teacher's behavior on particular examples. The student is never given any instructions on how to abstract from examples, nor is the student given any language for expressing what has been learned. It is hard to learn what one cannot express. But now we can express it!* (Sussman, 2005)

Not only can we express "how to" with computers – we can change it! Or at least we're in a better position to try.

As a thought experiment: imagine the distillation of the world's mathematical knowledge into a computer program that could coach students through difficult learning tasks in mathematics in an efficient manner. With a few adaptations, this program might also serve as a creative partner to professional mathematicians. With further adaptations, it may be able to come up with and prove interesting theorems on its own. The development of such a system would certainly constitute a change in mathematical culture. It would also have significant impact on in the way mathematical thinking is applied, in fields ranging from logic to logistics.

In order to move in this direction, one approach is to set aside the AI aspects at first, and focus instead on building a useful collection of *content*. The computer would be used primarily as a communication tool and database. As time goes by, additional sophistication can be added, expressing the user community's collective wisdom about what works and what doesn't. A commons-based peer production system is a suitable *context* for exploring cultural adaptation of this sort. The idea is that a software system created together with its users can be used to build not just a description of mathematical culture, but an actual working model that can serve as an engine for change.

Theory. A 6-month pilot study as a participant observer at the Peer-2-Peer University helped develop a general picture of the way learning and adaptation works (and sometimes fails to work) in peer production communities. For example, people often learn computer programming concepts and techniques while building free/open source software and participating in discussions on a developer mailing list. *Paragogy* aims to be a comprehensive theory of peer produced peer learning, and a qualitative complement to Benkler's theory of peer production.

Tool. In collaboration with colleagues at in the Knowledge Adaptation and Reasoning for Content (KWARC) group at Jacobs University, PlanetMath's custom code from 2001 was replaced with a new platform built using contemporary technologies. The aim was not just to modernize PlanetMath, but to build a reusable and extensible platform, which we called "Planetary" (Kohlhase et al., 2011). The particular instantiation of this system on PlanetMath is designed to support peer learning according to the paragogical understanding.

Test. Launching the Planetary system on PlanetMath constitutes a major change for users. The real-world deployment of the system gave the opportunity to study the response to this change, supported by focus groups and interviews. PlanetMath users provided concrete suggestions on how to extend the software model, which helps build a picture of the way they think about mathematics, and which also contributes to validating the paragogical approach to design, learning, and co-production.

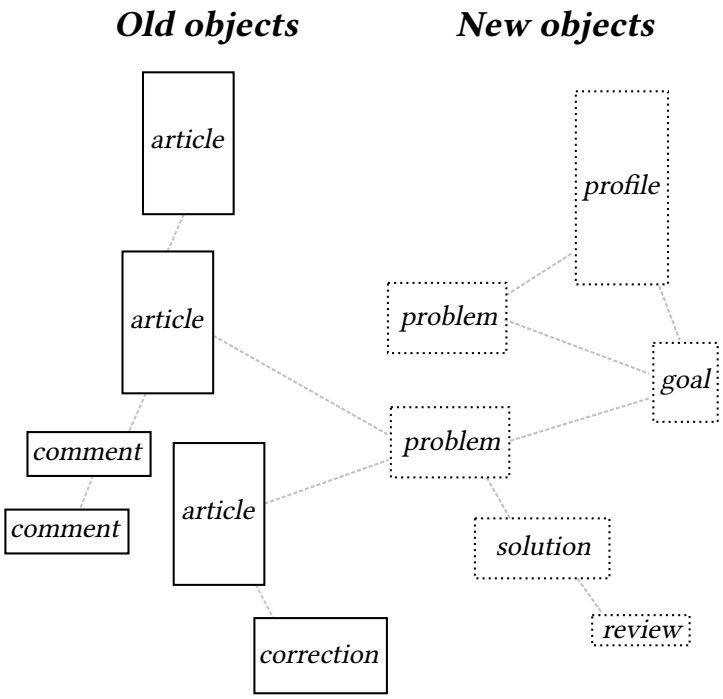


Figure 1: *This is the sort of extension we'd like to develop*

1.5 Trajectories associated with this work

Advances in method and theory. Using a commons-based peer production system as a way to study and express culture it is an example of the kind of self-conscious experiment with representation found to be lacking by Geertz (1973, fn. p. 19). This approach may be of interest to contemporary cultural anthropologists, who are increasingly involved in participatory research, and fieldwork in networked settings. The relationship between peer production and peer learning is explored through applied experimentation, not through strategies of description and argumentation that would be typical of work in Science and Technology Studies, a field this work does not aim to contribute to; *cf.* (DeLanda, Protevi, & Thanem, 2005). Although the methods are inspired in part by anthropology and philosophy, the outcome is a new computational account of mathematical collaboration that should prove useful in the design and evaluation of agent-based simulations, as well as online learning environments. The understanding of mathematical social behavior that is developed may be of interest to philosophers of mathematics. These contributions to method and theory are supported by practical advances in learning design and assessment, and provide a framework within which to assess claims about online mediated peer learning, as detailed below. These aspects may be of immediate interest to educators, as well as to education theorists.

New designs for peer learning. Gráinne Conole's (2008) perspective on "*learning design*" is that it is best understood as a way to create and represent practice. The design

we are using (Figure 1) will include all of the authoring and editorial practices associated with building and maintaining PlanetMath's encyclopedia – and a new layer for problem solving. The first layer features legacy activities like writing and editing articles, forum discussions, corrections and requests for articles on new topics. The second layer is new, and includes support for setting and solving problems, connecting problems and encyclopedia articles, discussing and reviewing problems, and creating course packets that combine problems with expository texts. From the user's perspective, the environment should offer some significant advantages over a standard textbook or problem archive. For example: nothing is more daunting than being faced with a problem to solve and not knowing what the terms in that problem mean. On PlanetMath, we will be able to provide automatically-generated links to the definitions of technical terms in problem statements. In the event that the linked material is not enough, and the user will be able to make annotations asking help. Answers to these questions will help improve the quality and relevance of the encyclopedia. Rather than having textbook problems serve as a daunting obstacle to application or research, they should, in this context, serve as stepping stones to relevant and meaningful engagement in mathematical practice.

A new technique for measuring learning efficacy. How can we validate claims about learning quality? John Anderson, Lynne Reder, and Herbert Simon advanced a consumer protection perspective on this question.

> *We should make a larger place for responsible experimentation that draws on the available knowledge–it deserves at least as large a place as we now provide for faddish, unsystematic and unassessed informal "experiments" or educational "reforms." We would advocate the creation of a "FEA" on analogy to the FDA which would require well designed clinical trials for every educational "drug" that is introduced into the market place.* (Anderson, Reder, & Simon, 2000)

Techniques for evaluating the efficacy of learning strategies and interventions can also be immediately relevant to users and system designers, in real time. Feedback on learning relevance could help users improve their problem solving skills. Chapter 6 draws on a recently developed approach from computational statistics to address these issues.

New frontiers for peer produced peer learning. Partly inspired by the idea of paragogy, Howard Rheingold convened the Peeragogy project,[*,†] which I subsequently joined as a contributing author, and later, editor. Rheingold positions the *Peeragogy Handbook* as "*a resource for any group of people, who want to learn any subject.*"[‡] This thesis will help to refer these claims to sound theoretical and empirical foundations.

[*] HTTP://DMLCENTRAL.NET/BLOG/HOWARD-RHEINGOLD/TOWARD-PEERAGOGY

[†] HTTP://PEERAGOGY.ORG

[‡] HTTP://WWW.YOUTUBE.COM/WATCH?V=SDUSPOUTYJE

1.6 Terminology

In cases where it is necessary to distinguish between the "original" PlanetMath built by Aaron Krowne and the "new" PlanetMath developed in the course of this thesis work, I will refer to them as PlanetMath/Noösphere and PlanetMath/Planetary, respectively.

Paragogy is the name for the theory of peer produced peer learning that is developed in this thesis, drawing on earlier collaborative writings with Charles Jeffrey Danoff, and, perhaps confusingly, on contemporary collaborative work in the Peeragogy project. Peeragogy is Howard Rheingold's neologism, adapting my earlier usage, chosen because this term is relatively self-explanatory. What "paragogy" might lack in directness, it makes up in nuance. A term that causes people to stop and think helps problematize the idea of peer learning. The term evokes but also challenges the idiom of the "guide on the side." It is a preexisting word in both English and Greek, so it brings additional associations through its etymology.* There are models of peer learning that remain "*provisionist*" rather than peer produced (Boud & Lee, 2005): in other words, the opportunity for peer learning is provided by some group, and taken up by others. Paragogy comes with a built-in immunity to provisionist thinking. As Chapter 3 explains, the emphasis in paragogy is on co-creating the context in which learning and production take place. The term *peeragogy* is most easily read as a synonym for peer learning. In this document, *Peeragogy* will always refer to the Peeragogy project, and its creations and namesakes (the *Peeragogy Handbook*, Peeragogy patterns, the Peeragogy Accelerator, *etc.*). I use *paragogy* as a shorthand for "the theory and practice of peer produced peer learning."

1.7 Related work

The concept of *emergent design* was applied by David Cavallo to create learning environments together with their users (Cavallo, 2000A, 2000B, 2004). Cavallo (2000B) describes his work with Thai villagers to help design local applications for the Logo computer system as "*applied epistemological anthropology*." More recently, this theme was taken up by Cavallo's PhD student, Arnan Sipitakiat (2007), in work with children learning robotics. This practice aims to learn along with research subjects as they build on their existing knowledge and skills, typically in *constructionist* scenarios – in which "*everything* [is to] *be understood by being constructed*" – as opposed to "*instructionist*" scenarios that emphasize teaching (Papert & Harel, 1991).

The idea of emergent design will be connected, in the current work, to the possibility of a "*problematic*" understanding of reality, after Deleuze (1968). Deleuze calls this the philosophy of difference, and it is grounded in his thinking about mathematical dy-

*Greek: *παραγωγή*, meaning "production", "generation"; English: "coaptation", *i.e.* "the adaptation or adjustment of parts to each other", and also "letters added for emphasis or to change the sense of a word."

namics. Building on the associated nexus of philosophical and scientific ideas, Manuel DeLanda (2011) makes a survey of the history of life through a series of computer simulations of dynamical systems, starting with water and ending with pyriamids.

The array of "*man-made information sources for the directive ordering of human conduct*" (Geertz, 1973, p. 251) that are relevant to mathematical practice can be treated with a similar multi-layered computational approach. Much more so than Geertz, the associated idea of *cybernetics* is associated with the work of the anthropologist Gregory Bateson, who insisted on the importance of not separating the "*internal*" and "*external mind*" (Bateson, 1972, p. 470). Ingold continues this line of thought and treats "*organism plus environment*" as a "*developmental system*" (Ingold, 2000, p. 19). Ingold does not talk about development using the language of difference from Bateson and Deleuze but emphasizes *growth* and, especially, *correspondence* (Ingold, 2013, pp. 21, 107). Chapter 4 will draw on Bateson's perspectives on learning.

Economics and organizational theory are almost entirely in the background in the current work: the focus is on phenomena that economists tend to treat as *externalities*. Briefly, one way to deal with externalities this is to create new markets, and another is restructure the firm; both scenarios were examined by Coase (1960, 1937), who emphasized the costs associated with exchange or transformation. Benkler's (2002) idea of commons-based peer production is a third model built around positive externalities that coalesce into new shared resources. Ostrom's (1990) design principles for the governance of common pool resources can be related to Beer's (1979, 1981) Viable System Model, which describes a cybernetic approach to organization.* Paragogy is developed as a parallel framework, focused on the dynamics of learning and adaptation.

Mathematics is usually managed through professional societies, firms, and the guild-like structures of academia. The possibility of managing it in a more "*open*" fashion has been explored by various authors; some, like (Buswell et al., 2004), considering technical issues, others focusing on social engineering (Pitman & Lynch, 2014). To my knowledge there is no prior study on considerations related to learning mathematics the commons-based peer production way. Within the field of mathematics education, most research focuses on classroom settings (Cole, 1998; Cobb, Wood, & Yackel, 1993; Cobb, 1989).

There has been broadly related research on learning in Wikipedia and Wikiversity (Lawler, 2005, 2011), and on learning in free/libre/open source software (Meiszner, 2010). I will draw on the latter in Chapter 4. It is also worth mentioning the traditions of *collaborative learning* (Bruffee, 1984), and *networked learning*.† The latter takes inspiration from Ivan Illich and Christopher Alexander. I draw on Alexander's design pattern methods, connecting them to the idea of emergent design and Deleuze's philosophy.

*http://p2pfoundation.net/Cybernetics_of_the_Commons

†http://en.wikipedia.org/wiki/Networked_learning

1.8 Summary of the argument

Part 2 of the thesis describes the research domain.

I make a case for using a peer production model to build learning resources for mathematics. I also make it clear that PlanetMath needs a new direction if it is to have contemporary relevance. I introduce paragogy through a description of the case study at P2PU in which it developed. The key theme in paragogy is to combine learning at the individual level with contextual change, that is, change in organization or environment. It is a theory of learning and adaptation. I develop paragogy as a framework, which is then used to organize the research literature that is relevant to peer produced peer learning in mathematics. This yields a preliminary set of criteria that can help address the main research question.

Part 3 describes the research methodology and preliminary studies.

A key philosophical idea in the thesis is the idea of emergence: the idea that the whole is always more than the sum of its parts. My methodology is rooted in Manuel DeLanda's ideas about emergent dynamics that develop within and across different domains and different levels of analysis (DeLanda, 2006A, 2011). At the micro-level, I develop a statistically-based study of the social antecedents to learning in PlanetMath/Noösphere. This shows how a system like PlanetMath could be used as a research instrument to study patterns of change from a quantitative perspective. I gather ideas about macro-level systemic change from representatives of three important stakeholder groups: students, mathematicians, and researchers who study mathematical practice.

Part 4 presents the primary results and discussion.

I describe the implementation of the new Planetary system, which has been built taking the ideas from stakeholders on board. After the system was deployed on PlanetMath, a user evaluation study was conducted. Based on material drawn from interviews with users about their experience with PlanetMath/Planetary, I develop a revised design that better reflects the purposes that users see the site as serving. Finally, I conclude that a paragogical approach to emergent design can be effectively applied within mathematics as a way "*to pose the problems set within it and to it by the differential relations it incarnates*" (Deleuze, 1968, p. 235).

Part 2
The research domain

2 Historical Background

Could learning mathematics become more like the learning that happens in the course of developing free/open source software?

2.1 The PlanetMath Encyclopedia

Beginnings

The pre-history of PlanetMath was tied to the fate of a similarly-named website, MathWorld.* Eric Weisstein began collecting the material now found in MathWorld as a high school student, and continued the project as a college student in the late 1980s. "Eric's Treasure Trove of Mathematics," went online in 1995, when Weisstein was a graduate student in astronomy at the California Institute of Technology.†

In November 1998, Weisstein made a deal with the CRC Press to publish his encyclopedia in book format, as the *CRC Concise Encyclopedia of Mathematics* (Weisstein, 2003). One year later, Weisstein accepted the position of Encyclopedist at Wolfram Research, Inc., and the newly-rebranded "MathWorld" site was unveiled in December 1999.‡ In March 2000, CRC Press sued Weisstein and Wolfram Research for copyright violation, forcing MathWorld off of the internet (US Patents Quarterly, 2000). Eric Weisstein was understandably dismayed: "[I]*f you ever assemble a body of knowledge that you want to share with others, you don't want to go through what I have just gone through.*"§

*http://mathworld.wolfram.com

†http://www.echarcha.com/forum/archive/index.php/t-19516.html

‡http://en.wikipedia.org/wiki/Eric_W._Weisstein

§http://www.echarcha.com/forum/archive/index.php/t-19516.html

MathWorld users Nathan Egge and Aaron Krowne, then undergraduates at the Virginia Polytechnic Institute and State University, were also dismayed – and in Autumn, 2000, they devised the idea for PlanetMath: it would be a collaboratively created mathematics reference work that would have resistance to copyright threats built-in, in the form of an open content license.

By Summer 2001, the basic infrastructure for creating an encyclopedia was complete, and a fledgling community had grown up around PlanetMath. It was not alone: Wikipedia also launched that year. Mayo Fuster Morell describes 2001 as a watershed year in the history of online creation communities – the year when the focus spread "*from free software to free culture*" (Morell, 2011).

The CRC lawsuit was settled for an undisclosed sum in late 2001, and on November 6, 2001, MathWorld returned to the internet.* But in the mean time, a new online community had been born – with some very different principles and practices. Whereas MathWorld's terms of use disallow archival copies, PlanetMath regularly published snapshots of the content for download. Moreover, under the terms of the site's license (originally the GNU Free Documentation License, later the Creative Commons Attribution-ShareAlike license)†,‡ users are permitted – and, indeed, encouraged – to copy, mirror, redistribute, print, remix, and reuse PlanetMath content for commercial or other purposes – so long as all such works are published under the same license as PlanetMath, granting downstream users the same rights.

Stabilization

The key reference on the founding and foundation of PlanetMath is Aaron Krowne's (2003a) Master's thesis, *An Architecture for Collaborative Math and Science Digital Libraries*, written at Virginia Tech under the supervision of Ed Fox. In this thesis, Krowne describes how the early design and development of the site benefited from continuous feedback in the `#math` IRC channel on Undernet.§ He also details the key technical and community features of the site as they developed in this period:

- A state-of-the-art system for displaying mathematical notation on the web, starting from LaTeX sources.
- A flexible authority model that can support both wiki-style articles (that anyone can edit), and a more academic style, where articles are owned by one person, who may, if they wish, grant co-authorship permissions to chosen others, and who

*http://mathworld.wolfram.com/about/faq.html#history
†http://www.gnu.org/licenses/fdl-1.3-faq.html
‡http://creativecommons.org/licenses/by-sa/3.0/us/
§irc://irc.undernet.org/math

must respond to separate commentary from peer reviewers in order to maintain ownership (Krowne & Bazaz, 2004).

- A discussion forum attached to every encyclopedia article, which helps give the resource its "*pedagogical slant*."*
- An autolinking service that helps integrate content into the site, by enabling authors to focus on the contents of one article at a time.
- The Mathematics Subject Classification (MSC) devised by the American Mathematical Society and Zentralblatt MATH provides a hierarchical browsing mechanism.
- A peer review workflow built around a content-level bug reporting feature called "corrections", and user-messaging feature called "watches." The workflow includes a feature whereby articles are "orphaned" if a correction is not responded to after a given period of time. A system for bug reports is particularly important in light of the authority model mentioned above: the historical PlanetMath is *not* an "*encyclopedia anyone can edit*"† in the typical wiki sense.
- "Attachments" expand individual articles with detailed proofs and related expositions.
- A scoring feature provides a rough estimate of how much value each user has contributed to the site.
- Downstream use and modification of PlanetMath's content is permitted, under the terms of the Creative Commons Attribution-Share Alike license (CC-By-SA), a copyleft free content license.

One can readily spot ways in which the core ideas of commons-based peer production – *modularity*, *granularity*, *integration* (Benkler, 2002) – had an influence on the design of the site. Articles are modular and relatively self-contained. Comments and corrections are even more fine-grained than articles. The Mathematics Subject Classification, attachment feature, and autolinking service connect related articles, assisting with integration. Article ownership modularizes control, and provides a protectionary measure for authors that is not given by the content license. Raymondite "homesteading" was built in to PlanetMath's custom software, Noösphere, from the beginning – and the system takes its name from Eric Raymond's (1998) essay.

In 2003, PlanetMath incorporated, and in 2005, obtained non-profit status, so that it could accept tax-deductible donations in the US. Together with a small stream of advertising revenue, donations covered hosting and other maintenance costs since that time.

*HTTP://PLANETMATH.ORG/HISTORY.HTML

†HTTP://EN.WIKIPEDIA.ORG

Mathematics problems

This section describes my own early involvement with the PlanetMath project.

From 2002-2004, I was enrolled as a graduate student in mathematics at the University of Texas in Austin, and in possession of a large and growing personal collection of tersely-written definitions and proofs related to the department's preliminary exams. For example:

(lebesgue outer measure: fact: lebesgue outer measure is infimum of lebesgue outer measures of open supersets)

1: $X \subset \mathbf{R}^n$

2: $L = \{O \subseteq \mathbf{R}^n : O \supset X\}$

3: $|X|_e = \inf_{O \in L}(\{|O|_e\})$

My name for this collection was the "Austin Problems in Mathematics – Cross-Index", named following the pattern established by *Berkeley Problems in Mathematics* (De Souza & Silva, 2004), and styled APM-Ξ.* In fact, this work had as much to do with the tradition of computer mathematics in the air in Austin as it had to do with exams.

This tradition was embodied in systems and projects like QED (Boyer et al., 1994), Maxima (Joyner, 2006), ACL2 (Kaufmann & Strother Moore, 1996), AM (Lenat, 1976; Lenat & Brown, 1984), Cyc (Lenat, 1995), ISAAC (Novak, 1976; Novak & Bulko, 1993), and KM (Clark & Porter, 1999) among others. Although I was in the mathematics department and not the computer science department, I was quite drawn to this body of work. My notion was to express all of the definitions and theorems relevant to the prelim exams in a symbolic form that could be relatively easily translated into computer code.†

I was convinced that writing down all of the relevant definitions, theorems, and proofs in meticulous detail would effectively guarantee that I would be able pass the exams – even if I wasn't able to build a mathematical artificial intelligence that would pass them on my behalf (or tutor me) in time. "Relevance," to my mind, was determined by

*HTTP://METAMESO.ORG/~JOE/MATH/XI.PDF

†Per HTTP://METAMESO.ORG/~JOE/MATH/100.PDF, the translation of the above example into a more computer-friendly syntax would have been:

```
(defthm leb-outer-measure-inf-over-open-supersets (X)
  (subset X Rn)
  (let ((L (setof (open-subset O Rn) :st (supset O X))))
    (eq (leb-outer-measure X)
        (inf :over O :in L :of (leb-outer-measure O)))))
```

inclusion among the problems and ideas found in old exams – which were available from the department's filing cabinets (without solutions).

I purposefully neglected the fact that the required first year courses were expressly designed to prepare students for *this* year's preliminary exams, which would be written by the instructors for these courses. Accordingly, course homework and in-class exams would have been a better source of material for my cross-index. But here I found myself caught in something of a perfect storm. Compared with my earlier experiences with more tutorial-style instruction as an undergraduate, the almost exclusively lecture-oriented classroom culture was stifling. I had been further spoiled by early participation in collaborative mathematics research. I thought that the instructors were supposed to help me learn mathematics, and indeed, help me to get to the research frontier as quickly as possible; instead, it seemed as if they were putting in effort to make the entire experience as *difficult* as possible. Finally, I was aware that other students would often study for the courses together, but I wasn't sure what to think about this. Was it encouraged? Was it cheating? As it turned out, I wasn't invited, and I didn't invite myself.

Instead, I spent most of my time building my cross-index of old exam material, reading philosophy books, and learning how to program in LISP with support from the contributors to mailing lists related to GNU Emacs, "*the extensible, customizable self-documenting display editor*" (Stallman, 1981). And I thought: why stop with the graduate curriculum? And so the goal shifted: let's build a copyright-free computer simulation of all known mathematics!

It seemed to me that the mathematics community was entirely uninterested in any potentially liberatory aspects of computers. A new project focused on making computer mathematics relevant to human mathematicians – here I was inspired by GNU and Deep Blue – seemed to be the only suitable response. I called this the *Hyperreal Dictionary of Mathematics* project; *cf.* (Baudrillard, 2002).[*,†]

In 2003, I discovered the PlanetMath website and initiated a correspondence with Aaron Krowne about the potential for overlap between PlanetMath and HDM. I admired the fact that he had put together a real, working, system, with actual users. Considering the magnitude of HDM project, I knew it would eventually need a crowdsourcing component. PlanetMath wasn't doing quite what I had in mind, but it seemed like a good place to start. In Autumn of that year, we uploaded the contents of the APM-Ξ into the PlanetMath encyclopedia as world-editable "seed entries".

This was not received particularly favorably by PlanetMath users, which generated a range of useful reflections.[‡] The primary concerns raised by community members were:

[*] HTTP://WIKI.PLANETMATH.ORG/CGI-BIN/WIKI.PL/HDM

[†] Kellner (2007) offers a useful overview of Jean Baudrillard's writing.

[‡] HTTP://WIKI.PLANETMATH.ORG/CGI-BIN/WIKI.PL/ONE_WEEK_IN_OCTOBER

(1) The entries could not be understood without reading an accompanying FAQ, and were not "human readable";

(2) A casual visitor to the PlanetMath website might get the wrong impression about the nature of the encyclopedia when looking at the owned by the user apmxi; and,

(3) Nearly 600 entries had been introduced into PlanetMath by the site's administrator in one big batch, circumventing, at least in outward appearances, the site's usual model of careful review and collaborative editing of entries.

Subsequent to a poll, it was decided that the entries owned by apmxi would be "orphaned," and any that were not adopted by community members after a week would be deleted from the encyclopedia. This was the fate that befell most.

It was clear in hindsight that the APM-Ξ material – independent of its potential value for computer mathematics – was *not* something that had a positive value for most people in the PlanetMath user community. Although it was something of a personal embarassment, the apmxi incident was a testament to the strength of PlanetMath's norms, and it illustrated the role that shared norms play in maintaining an integrated common resource. It also showed that the specific affordances of computer technology, for instance, for bulk processing, or for using hypertextual methods to present alternate treatments of a given topic, would need to be used and developed carefully, so as to work well for the *people* involved.

As could have been expected, over the next year, things ground slowly to a halt for me in Texas. With no prelim exam success, I was transferred into the Master's program, and given the opportunity to work with the mathematician and computer scientist Robert Boyer, but he and I couldn't see eye to eye about a thesis topic. At the end of Autumn in 2004, I was formally asked to leave.

Although I had been an errant and irresponsible mathematics graduate student, I had assembled another identity along the way: that of a necessarily interdisciplinary researcher working on understanding and transforming mathematical culture. Although PlanetMath was in many respects "marginal" to mainstream mathematics, and my project was on the fringes even of PlanetMath, this was a digital library – where the margin was perhaps not such a bad place to be.*

*In connection with the idea of "*math for the people, by the people*", is worth pointing out that Arendtian *constituent moments* "*enact felicitous claims to speak in the people's name, even though those claims explicitly break from the authorized procedures or norms for representing the popular voice*", *cf.* (Frank, 2009, p. 210).

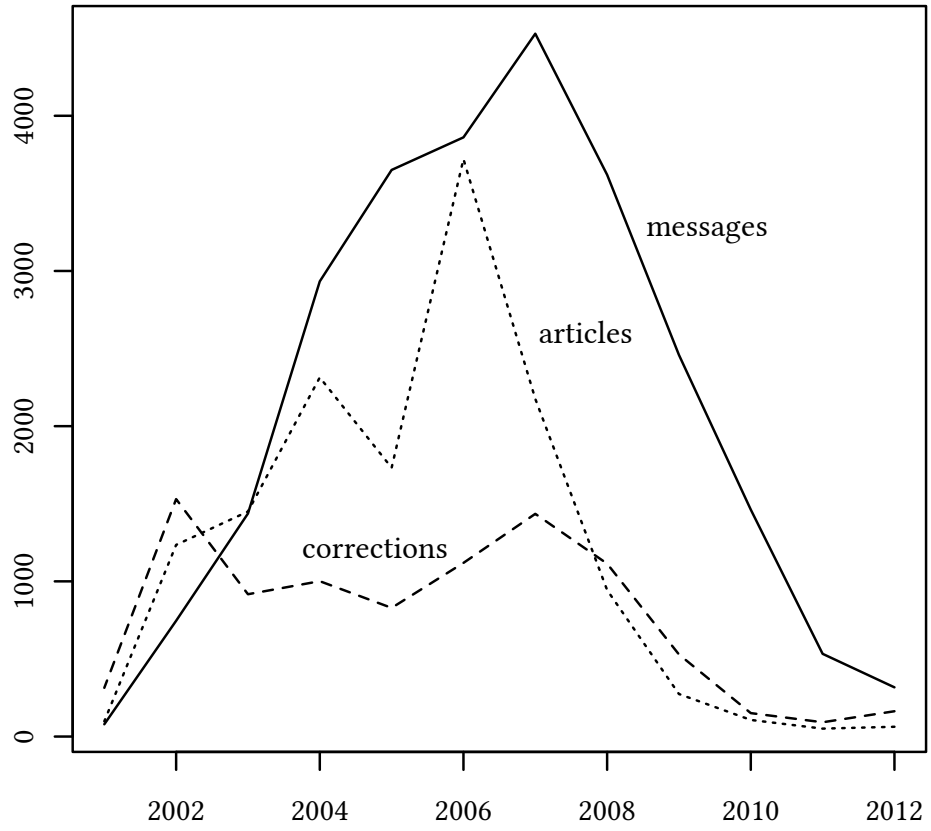


Figure 2: *Plot showing the number of messages, articles, and corrections added to PlanetMath over the last decade*

Reach and scope

Since PlanetMath describes itself with the by-line "by the people, for the people," one way to assess its quality, success, and vitality is through data on user participation. As an initial foray, Figure 2 shows the contributions per year in various sectors of the site. As is typical for online communities, contributions are uneven, and roughly follow a "power law" distribution, *i.e.* $f(x) = ax^k$. The relevant exponent is approximately -4, which represents a much steeper dropoff than popularity on the internet (≈ -3), energy efficiency of scale for living organisms (≈ -2), or word frequencies in natural language texts (≈ -1); *cf.* (Corneli, 2011).

PlanetMath can also be evaluated as an encyclopedia. A brief summary, following the framework used by Emma Previato (2004) in her review of the *CRC Concise Encyclopedia of Mathematics* (Weisstein, 2003), gives a look at how PlanetMath measures up (Table 1).

Coverage	The median entry would be an advanced undergraduate or beginning graduate topic. PlanetMath is generally considered to have more in-depth treatment of technical issues, e.g. of proofs, than that found in Wikipedia. Over 15000 terms are defined in over 9000 articles.
References	Present in many articles, although there is not yet a unified database of references or style of presenting them (this is planned).
History	600 items, mostly 20th Century or later (HTTP://PLANETMATH.ORG/MSC_BROWSER/01Axx)
Audience	Consistent with the coverage, there have been 4127 posts in the "Graduate/Advanced" forum, 4261 posts in the "University/Tertiary" forum, and 1199 posts in the "High School/Secondary" forum.
Clarity	There is no hard and fast rule. Some articles will tend to be minimalistic, but precise (particularly when the authors have English as a second language). Other articles even from native speakers may be verbose and vague. In any case, debates over clarity of presentation are intense, and a high standard is maintained.
Pictures	There are over 600 images, but since this is only about 7% of the number of articles, the prevalence could be improved. A unified database/gallery of pictures would help.
Accuracy	At the time of this writing there are 48 outstanding corrections; more than 14080 corrections have been filed since the site began, though 2337 are classified as "addenda", meaning that no mistake is implied, and 9182 are classified as "meta/minor", which suggests that in the entire history of PlanetMath some 2561 real errors have been found through the peer review process (and all but a few fixed!). Note that these numbers do not take into account changes initiated by authors without prompting, and would tend to under-represent error fixing in world-editable articles.
Unusual	PlanetMath provides "math for the people, by the people".
Weight	PlanetMath can be comfortably edited from a lightweight laptop weighing about 1kg (and browsed from a mobile device weighing considerably less).

Table 1: *Succinct review of the PlanetMath Encyclopedia*

The dropoff in participation in recent years is presumably due to a mixture of factors: saturation of the encyclopedia, technical problems with the site's software, and the rise of successful competition. Most notable among the "competitiors" are two other sites, Wikipedia and StackExchange, both using the same license as PlanetMath.*,† It is clear from Figure 2 that PlanetMath will need a new approach if it is going to continue to be a productive community.‡

As Sophia Angeletou et al. write:

> *Health of online communities is a relatively new and complex concept that is codependent on the emergence and evolution of user behaviour in those communities.*
> (Angeletou, Rowe, & Alani, 2011)

2.2 The broader context: Other candidate solutions

There are several well-established *gratis* (and some *libre*), players in the open/online mathematics learning space. In addition to Wikipedia and StackExchange, mentioned above, other relevant service providers include MITx, Coursera, OCW, edX, Stanford, Khan Academy, PurpleMath.com, OpenStudy.com, and functionspace.org. On offer are articles, Q&A forums, courses and course materials, problem sets and videos, as well as peer support in various forms.

For now, through it's economy of scale, Wikipedia is dominating in the encyclopedist prosumer market; *cf.* (Jankowski, 2013). After more than a decade of work, PlanetMath is valuable as a highly curated topic-specific reference resource, but Wikipedia is more active. It also seems to define many more mathematics terms than PlanetMath, although a structured comparison of coverage is still being charted.§ Perhaps surprisingly, even Wikipedia is less active than it once was (Simonite, 2013), and there appear to be bigger patterns at work, with social media on the rise, while centralized peer production sites are less used.

Could PlanetMath have any technical, or ethical, advantage (Landini, 2012) over existing technical innovations like Wikipedia, Massive Open Online Courses (MOOCs), Q&A sites like StackExchange, and videos collections and problem sets like the ones produced by Salman Khan and his colleagues? Is there already an adequate answer to the demand

*"*There are approximately 29,280 mathematics articles in Wikipedia,*" from HTTP://EN.WIKIPEDIA.ORG/WIKI/PORTAL:MATHEMATICS

†As of June 16, 2013, math.stackexchange.com and MathOverflow contain 147768 and 42735 questions, respectively.

‡Note that hundreds of PlanetMath articles have been used as seed material for world-editable articles in Wikipedia, under the auspices of the (somewhat inaccurately named) "PlanetMath Exchange". HTTP://EN.WIKIPEDIA.ORG/WIKI/WIKIPEDIA:WIKIPROJECT_MATHEMATICS/PLANETMATH_EXCHANGE

§Deyan Ginev, personal communication

for high-quality, low-cost, globally accessible, mathematics instruction? This brief discussion focuses particularly on comparisons with Khan Academy.

Khan Academy,[*] founded by Salman Khan in 2006, is built around a collection of instructional videos hosted on YouTube, which are supplemented with work-book style exercises, mostly for school-level topics.[†] It was initially best known for mathematics content, although more recently, it has added content in a range of other fields.

Khan Academy has helped to popularize the "flipped classroom" model, in which lectures are viewed out of class, and homework is assigned to be done in class (Khan, 2012). The key thought behind this model is that teachers can help students who are struggling, or use class time to invite students to help one another. Adopting a flipped classroom model for graduate-level mathematics would have helped address my discontents at UT Austin, since in-class discussion would have replaced note taking as the norm.

However, Khan Academy and most current MOOCs still have a highly-centralized production model, whereas PlanetMath's content is entirely user-built. Khan Academy's application of the non-free Creative Commons Attribution-NonCommercial-ShareAlike license (CC-By-NC-SA) reflects their centralized stance.[‡,§]

PlanetMath can continue to grow in a comparatively demand-driven and participatory manner, which facilitates the development of advanced material and detailed expositions. To use Landini's (2012) term, PlanetMath is considerably more "*malleable*" than Khan Academy and most MOOCs. Public wikis provide another relatively malleable alternative – but they are most malleable at the content level. In fields like sociology, it makes good sense to invite students to author or edit encyclopedia articles as part of the learning process (Konieczny, 2007). It is hard to imagine a mathematical curriculum that did not focus on problem solving. A customized version of a mainstream content management system, together with robust support for LaTeX, makes Planetary at least as flexible a typical wiki, and can help build additional workflows.

While a centralized production model makes a great deal of sense for a resource with highly integrated content (like a course), a distributed production model makes more sense when building a resource that is relatively disaggregated (like a Q&A collection). PlanetMath/Planetary allows for both disaggregated one-off questions and answers, and more cohesive textbook or course-like exposition – and it seeks to link these two models together. As with Khan Academy, the new PlanetMath content should be useful for self study, which may facilitate more efficient use of in-person interactions. And like MOOCs, PlanetMath should also simultaneously open up new avenues for peer support

[*] http://www.khanacademy.org/

[†] https://www.khanacademy.org/exercisedashboard

[‡] http://creativecommons.org/licenses/by-nc-sa/3.0/

[§] http://freedomdefined.org/Licenses/NC

online. Unlike these more centralized models, PlanetMath is oriented around end-user co-production of structure, preferring an "emergentist" approach to what Boud and Lee (2005) would call the "*provisionist*" approach.

In the first place the question is not whether this approach is "better," but what it may possibly be better for. I would argue that an emergentist strategy is better for working with advanced content. Only relatively well-understood content can be developed locally, for any given locale. This suggests a deeper point, which does indeed take on an ethical dimension. In short, we should eschew provisionist approaches to mathematics on the basis of Euclid's assertion that *there is no royal road to geometry*. Even if we managed to build an computer tutor that delivered superb personalized instruction, and which made learning what we currently think of as difficult mathematics fairly easy, this too should ultimately be surpassed. That would be, as ever, a lot of work. The most important insights would surely come from the system's users.

The fact that the courses and lectures provided as part of mainstream mathematics are good enough "for most people" is not a real counterargument, since, in the first place, most people have never set foot in an advanced mathematics classroom, and secondly, we don't yet know how effective an alternative emergentist approach would be. Existing educational research suggests that if they were available, personalized tutorials would be more effective than classrooms (Bloom, 1984). Unless there was compelling evidence to show that an emergent peer learning approach was either ineffective, or otherwise impractical, it seems vital to give it a serious try, at least on an experimental, demonstation, basis.* Emergence, here, does not mean adding more and more elements to a loose network of associations and expecting meaning to appear, and neither does it at entail giving up on training as such. Rather, it is a question of establishing a suitable *ethos*, as a higher-order constraint on behavior; *cf.* (Sloterdijk, 2013, pp. 162–163), (Deacon, 2003). Once this is done, the corresponding ethical argument is no longer needed: the point will have become, as mathematicians like to say, obvious (Cooksey, 2005, p. 3).

2.3 The road ahead

Although it has experienced both a rise and a fall, PlanetMath/Noösphere was reasonably successful as an online community: the software stabilized early on and has required little upkeep, while the site continued to grow. The downside of early stabilization is that relatively few new features were developed since the early 2000s. The sign that there might be a problem – and some unrealized potential – was that feature

*Research mathematics already has many of the features typical of peer production, and the standards for openness in modern mathematics tend to go beyond those in many other academic fields. For instance, Article 2 of the WIPO Copyright Treaty holds that mathematical concepts, per se, are not copyrightable. Mathematical expressions may be further excepted by the legal notion of *scènes à faire*.

requests kept coming in, while very little new code was being written.* Despite these concerns, there is still an active interest in the project nearly a decade and a half after its creation.

Figure 3 presents a use case for PlanetMath that inspires the technical work carried out in support of this thesis. The diagnostic features described in the use case were not realized, although in light of the new semantic features that are included in Planetary, they may not be far off. Certainly they would not require a mathematical "Deep Blue".

As with flipped classrooms, had a system like this existed in 2003, I would have been a very happy user, and contributor. My basic concern at the time was: couldn't learning mathematics be done the same way I was learning LISP? There was detailed documentation available in Emacs, together with the full source code, a Read-Eval-Print Loop (REPL) for trying things out, and an integrated debugger. If I ran into questions that I couldn't resolve on my own using these resources, there were public mailing lists with participants who were always glad to help me, and to suggest improvements to any new packages I might upload. Although much remains to be done, PlanetMath/Planetary should eventually support this style of learning in mathematics.

Indeed, the renewed activity in software development is based on a redesign, not just a rebuild. PlanetMath/Planetary puts more emphasis on content reuse, and, being using a mainstream content management system, makes the system better suited than Noösphere was to support ongoing development. In a philosophical sense, PlanetMath represents a new and distinct type of encyclopedia, and it is now poised to begin its final ascent to the literal classical meaning of *ἐγκύκλιος παιδεία* ("recurrent instruction"). Because the technology that supports the site is special-purpose (and is not constrained to be "a wiki"), it can helped zero in on the specific set of features that support commons-based peer production – and now peer learning – in mathematics.

2.4 Summary

From a development standpoint, this is an critical moment, since there are mature, extensible, web frameworks available, together with a relevant collection of content, together with a clear demand for a new approach. As a researcher, this is interesting, because it affords the opportunity to examine technological change as an insider in the process. At the same time, PlanetMath occupies a novel position: although it is host to various examples of mathematical practice, it is still outside of the mathematical mainstream. It is useful as vantage point from which to survey and reflect on mathematics as a whole.

*http://wiki.planetmath.org/cgi-bin/wiki.pl/Feature_Requests

Primary Actor: A student, Madeleine, who is trying to learn multivariable calculus.

Main Success Scenario:

1. Madeleine is enrolled in an advanced calculus course at university. She learns about PlanetMath from her instructor who recommends it as a place for extra practice with homework problems. Madeleine creates an account, fills in basic profile information, and starts solving problems that the system supplies based on the information she added.

2. The problems that the system supplies are automatically linked to reference resources in PlanetMath's encyclopedia. This expository material gives Madeleine easy access to the relevant mathematical concepts, examples, and hints needed for solving the increasingly difficult practice problems. However, she eventually runs into a problem where neither the automatically supplied information, nor her current knowledge of the subject, is sufficient. She's completely stuck on a problem having to do with water flow in a pipe! Madeleine attaches a help request to the problem: "I understand that I have to use the two variables x and y to solve for water flow, but I don't understand what the boundary limits of the equations would be: do I have to convert it to polar coordinates?"

3. This request is noticed by Natalie, a mathematics graduate student who regularly looks at the feed showing "recent requests for help with advanced calculus." She sees that the reference resources linked to Madeleine's problem are probably not sufficient, and that Madeleine's idea about using polar coordinates would work. Natalie makes some changes to the encyclopedia indicating that converting to converting to polar coordinates can be necessary in pipe flow problems, and sketches an example. Natalie then checks that this information links to Madeleine's problem correctly, and alerts Madeleine to the changes. With this new information, Madeleine is not only able to solve her problem, but can proceed with confidence: she had the right idea after all!

Figure 3: *An example use case for problem solving support on PlanetMath*

Although there are other platforms that people can use to discuss problems, none of them clearly present the opportunity to make something that is better than a textbook from the point of view of learning, cohesion, and exposition. Considering the difficulty of managing advanced material and personalizing the learning experience, I make the case for using a somewhat distributed peer production model, rather than a centralized provisionist model. Although there are many great textbooks, and great teachers, we don't yet know whether the computer, or computer mediated communication, can deliver comparable (or complementary) results. The kind of research that is developed in thesis is needed in order to make that research possible.

This project builds on a decade of experience with the PlanetMath community. In this chapter, I have highlighted some of the problematic aspects that experience. I came to the project with the view that mathematics itself is already a kind of simulation, even if mostly still carried out on paper and with chalk – and, indeed, already a kind of virtual community, even if most of its activities take place offline and in person. My philosophy at the time was "*To simulation we reply by simulation*" (Baudrillard, 1983, p. 87); these days, it's closer to "*What it is is up to us*" (Rheingold, 2000, p. 39). This applies to PlanetMath, certainly, and may apply to mathematics more broadly, if we're willing to tackle some problems that aren't drawn from filing cabinets (Deleuze, 1966, p. 15). To supplement my participant observer perspective on these matters, I've presented some figures on participation drawn from the PlanetMath database, which confirm the need for a change in direction if the PlanetMath project is to continue in an active form.

3 Theoretical Background

Theories of pedagogy can help us "*get to the root of conditioning through practising repetitions*" (Sloterdijk, 2013, p. 199) – but they fail to model the productive, explicating, way in which learning unfolds in online peer production communities. These features are better described with the new-old word, *paragogy* (from *παραγωγή*, production).

3.1 Paragogy

The term *paragogy* (literally, "para-" alongside, "-gogy" leading) is used here to characterize the critical study and practice of peer produced peer learning, adapting the classical concept of *pedagogy* and the relatively recent notion of *andragogy* (Knowles, 1968) to a peer learning context. The need for a theory of this nature was articulated in dialog and collaborative research with Charles Jeffrey Danoff (Corneli & Danoff, 2011a, 2011b). Having first met in person at Wikimania 2010, and then again online at P2PU where we were preparing to become course facilitators, we agreed to enroll in one another's first round of courses and share feedback. P2PU aims to provide a social layer that is not supplied by default with Open Educational Resources,* and our agreement presented us with another small scale social context within P2PU. Paragogy was developed in Danoff's *Collaborative Lesson Planning* course, as we tried to understand why my *DIY Math* course had gone so badly (see Appendix A for the course description and post mortem). We used paragogy to examine a range of courses and support experiences at P2PU. After describing this analysis, the chapter concludes with a discussion of paragogy as a unified perspective on individual and contextual change.

*http://www.shuttleworthfoundation.org/projects/p2pu/

DIY Math

During Autumn of 2010, I made a brief and fairly naive effort to run a discussion-based online mathematics course at P2PU called "*DIY Math*", with the hope that participants would self-organize a suitable peer-learning structure. My motivation for running the course was partly to experiment with peer learning and self-organizing structures in mathematics. At the same time, I was also interested in exploring possible organizational connections with P2PU, where I hoped PlanetMath might eventually come to play the role of a mathematics department. Table 2, drawing on a broader discussion of roles in education from (Corneli & Mikroyannidis, 2012), applies a framework from Ken Wilber (1997) to summarize the overall picture.

When *DIY Math* actually ran, participants quickly lost interest, and all participation stopped after two weeks.* Although this was clearly a failed experiment from a teaching perspective, it is actually quite useful as a baseline reading for the broader project of understanding peer produced peer learning. The experiment shows what happens in a learning project where there is essentially no structure provided at the outset.

The rapid failure of *DIY Math* provoked a certain amount of self-criticism, but it also led me to ask the following critical questions about the learning context at P2PU:

- *What if P2PU was taking an ongoing survey of the "wished for" course topics?* Then a future course organizer would be able to create courses specifically tailored to the interests of self-selected participants.
- *What if every participant specified their level of commitment in advance?* Building a "contract" with the facilitator – and with the community – would help ensure that people would be make appropriate commitments (and keep them).
- *What if facilitators themselves got more coaching and peer support?* Ongoing discussions in the P2PU community mailing list helped a little, but not enough. Quality control was sacrificed to free experimentation. However, it was not entirely clear how a shared knowledge base about "what works" would develop out of the individual experiments.

More than the bare-minimum structure will be needed for peer learning in mathematics to succeed. The next term, after polling people about their interests, I ran a second more traditionally structured and focused course ("*Math for Game Designers*"), with a detailed syllabus prepared in advance, and this course did not go much better in terms of student participation than *DIY Math*. Again, this suggests that it is not simply more structure but the *right* structure that is needed.

*https://groups.google.com/d/msg/diy-math/X8uCWYRFzVQ/ccLCATWEYvoJ

	An "ideal" participant in the DIY Math course:
I	say what I want to study
We	talk about difficulties and successes
Its	discussions on a shared mailing list
It	helps me learn mathematics (and improve my skills at being a self-directed and peer-to-peer learner)
	A P2PU course facilitator:
I	come up with a course I'd like to facilitate, and then facilitate it
We	discuss ideas about how our courses might work and what "facilitation" means (e.g. as opposed to "teaching")
Its	discussions on a community mailing list and other settings (including discussions with participants in the course as it runs)
It	helps me improve my skills at a course designer and facilitator (and it's fun talking about and practicing this stuff!)
	An a posteriori picture of a DIY Math participant:
I	try to figure out what to study
We	sometimes give or get advice that isn't always so helpful (and most of it isn't for me, anyway)
Its	a bunch of good intentions that lead nowhere
It	confirms my sense of the difficulty of learning anything in a self-directed fashion (and the difficulty of mathematics in particular)

Table 2: *Roles surrounding the* DIY Math *course at P2PU*

Paragogy principles

I devised and shared five "paragogical principles" with Danoff's *Collaborative Lesson Planning* course, where they were jointly revised. I chose andragogy (Knowles, 1968) as a starting point for adaptation, knowing that since andragogy is a theory of *adult education*, and the aim was a new theory of *peer learning*, serious alterations would need to be made. I made use of Laurie Blondy's (2007) critique, which shows that the following major assumptions of andragogy often fail in online mediated learning applications.

1. **That adult learners are self-directed.**
2. **That they bring a wealth of experience to the educational setting.**
3. **That they enter educational settings ready to learn.**
4. **That they are problem-centered in their learning.**
5. **That they are best motivated by internal factors.**

A paragogical view admits a mixture of self-directed, other-directed, dependent, and independent, behaviors, without saying that any of this is good or bad. It suggests that a particular sort of self-concept may be less important than a robust concept of "*shared context in motion*," which constrains but is also shaped by behavior, as per Kitaro Nishida; *cf.* (Abe, 1988). It emphasizes social intelligence over so-called "mental age," and focuses on social dynamics rather than individuals.*

A paragogical view says that, whatever our past experiences may be, we have a lot to learn about learning. In order to show why, it is again helpful to translate the key term: "*Μάθησις means learning; μαθήματα, what is learnable*" (Heidegger, 1977). On the strong etymological view, mathematics is the study of things insofar as they are learnable, and what makes something learnable is exactly what makes it mathematical. For Heidegger, this is the axiomatic. Deleuze uses a related but less logistic conception: the problematic. "*Learning is the appropriate name for the subjective acts carried out when one is confronted with the objecticity of a problem*" (Deleuze, 1968, p. 204); problems are themselves constituted in a supple and always "*meta-mathematical*" dialectic, understood to be "*a system of differential relations between genetic elements*" (Deleuze, 1968, pp. 203, 229).

At the same time, from a paragogical perspective, learning is not an automatic default; it is a specific sort of highly conditioned side-effect. One may be stuck in a rut. A *peer* should not be conceptualized as someone who is stuck in the same rut. Rather, people who learn together or from one another, who help one another get a better grasp of the world, are understood to be "peers."

The paragogue often has an application in mind, but learning "*always takes place in and through the unconscious*" (Deleuze, 1968, p. 204), and one problem leads to others. "*Learning to swim or learning a foreign language* [...] *tears us apart but also propels us into a hitherto unknown and unheard-of world of problems*" (Deleuze, 1968, p. 241).

Finally, realizing a motivation seems at least as important as its existence in the first place. Merely indicating that a given motivation is "internal" or "external" is not particularly incisive. More can be learned by realizing motivations where this is possible, then going on to the next thing.

Après andragogy, then, these are the five paragogical principles:

1. **Changing context as a decentered center.**
2. **Meta-learning as a font of knowledge.**
3. **Peers provide feedback that wouldn't be there otherwise.**
4. **Learning is distributed and nonlinear.**
5. **Realize the dream if you can, then wake up!**

* *Cf.* Eugene Matusov [§1.1, p. 3].

It should be elucidating to mention straight away that the new five-point framework can be understood as an extension of "I/We/Its/It" from the previous section. In these terms, the framework amounts to "Context/I/We/Its/It". Whereas andragogy deals with what Manuel DeLanda would call "*tendencies*," paragogy deals with – necessarily contextual – "*capacities*."

> *While tendencies make a list of essential properties look falsely permanent capacities explode it since for a given whole it would be impossible to list all the different ways in which it can affect and be affected by innumerable other wholes.*
>
> (DeLanda, 2011, p. 186)

Nonaka and Takeuchi (1995) drew on Nishida's thinking to build the SECI model of knowledge production.* This model has close parallels with Ken Wilber's (1997) "I/We/Its/It". It is curious that neither of these frameworks makes an explicit place for context, although Nonaka and Takeuchi would likely agree with Gregory Bateson that the key point is that "[i]*t is the context which evolves*" (Bateson, 1972, p. 164). In a more detailed interpretation, "Context" could be decomposed into nested, overlapping, or adjacent contexts. Context, thought of as an "*environment*," plays an important role in *constructivist* thinking and learning design:

> *Thinking of instruction as an environment gives emphasis to the 'place' or 'space' where learning occurs. At a minimum, a learning environment contains: (1) the learner; (2) a 'setting' or a 'space' wherein the learner acts, using tools and devices, collecting and interpreting information, interacting perhaps with others, etc.*
>
> (Wilson, 1996, p. 4)

Again, in the paragogical view, the environment is not thought of as simply "given."

> *Interactions observed in complex ecosystems need not be regarded as expressing self-organizing properties of the systems themselves; instead they can be understood as the consequences of the various and variable adaptive strategies of individual organisms living together in restricted spaces.* (Vayda & McCay, 1975)

Andrew Vayda and Bonnie McCay position adaptivity within a "*neo-Darwinian selection theory*," in which life is subject to various "*hazards*."

> *Important here is the notion of* ***process*** *of response, including processes whereby the unit of action may shift from individuals to various forms (and degrees of inclusiveness) of groups and perhaps back to individuals, in accord with the magnitude, persistence, and other characteristics of the hazards in question.*
>
> (Vayda & McCay, 1975), original emphasis

*SECI stands for *Socialization/Externalization/Combination/Internalization.*

The notion of *process* underscores to the critical importance of a context in which that process can unfold.*

Within the field of education, Marlene Scardamalia's notion of "*collective cognitive responsibility*" (Scardamalia, 2002) is sympathetic, although it remains basically provisionist. Her claim is that in the standard classroom model, "*all the higher-level control of the discourse is exercised by the teacher.*" Students are "*reactive*" and "*receptive*," their work focused on "*tasks and activities.*" Small groups and decentralized, constructive, computer mediated communication are seen as two possible alternatives that "*turn more responsibility over to the students.*"

3.2 P2PU: An "After Action Review"

The paragogy principles are intended to serve as guidelines for building successful peer learning experiences. Corneli and Danoff (2011A) looked at how each of principles was (or was not) implemented at P2PU. A summary of this analysis is presented below. In addition to the paragogy principles, the US Army's (2002) After Action Review (AAR) provided the basic technology for *doing* paragogy. The AAR's intended purpose is to "*identify strengths and shortcomings in unit planning, preparation, and execution, and guide leaders to accept responsibility for shortcomings and produce a fix.*" It places much more emphasis on learning at the group level than the corresponding basic technology of andragogy described in (Carlson, 1989). Although the context of application is different, there are parallels between the AAR and the Action Research cycle; *cf.* (Carr & Kemmis, 2003; Lawler, 2008). The four steps in the AAR are:

1. *Review what was supposed to happen (training plans).*
2. *Establish what happened.*
3. *Determine what was right or wrong with what happened.*
4. *Determine how the task should be done differently the next time.*

(US Army, 2002)

Changing context as a decentered center: Mapping system dynamics and semantics We organized multiple courses where participants were supposed to interact and learn about the subject matter: *Collaborative Lesson Planning* Fall 2010 and Winter 2011 (co-organized by Charles Jeffrey Danoff and Dr. Majorie King); *DIY Math* (Fall 2010), and *Math for Game Designers* (Winter 2011) by Joseph Corneli; *Open Governance and Learning* (co-organized by Corneli and Marisa Ponti); and, in Spring 2011, *Shaping P2PU*,† which was an "intervention" based on a preliminary version of the paragogical evaluation presented here. Due to critically low participation, the mathematics courses did

* *Cf.* Eugene Matusov [§1.1, p. 3] and Tim Ingold [§1.3, p. 7].

† HTTP://NEW.P2PU.ORG/EN/GROUPS/P2PU-THE-COURSE/

not run to completion. Participation in *Collaborative Lesson Planning* and in *Open Governance and Learning* was minimal, but sufficient for a conversation to be sustained for the entire 6 week session. The theory of paragogy was born in these discussions. More than 30 people signed up for *Shaping P2PU*, but participation was very low. These experiences were typical: participation within courses was globally uneven, and falling over time. Our best experiences as course organizers happened when we were committed to working through the material ourselves. Combining facilitator dedication with gentle prompts to participants to follow through on their commitments could go a long way towards keeping engagement at a reasonable level, but this only works when commitments are somewhat clear in the first place. The case of *Shaping P2PU* shows that organizer commitment is not enough. The P2PU ecology contains an implicit rubric for learning and engagement: from the time a member signs up for a course, to its completion, peers go through a cycle.* P2PU could implement more formal check points throughout the cycle, requiring participants to specify, reaffirm, or adapt their commitments in relationship to judgments about quality. The AAR could be useful in this regard.

Meta-learning as a font of knowledge: Transparency, accountability, and tone Support for facilitators was offered as a P2PU course (*Course Design Orientation*), in mailing lists, via weekly phone calls, in a Q&A issue tracker, and via other informal channels. Participants in courses were presumed to be ready and willing to contribute in a useful fashion. The core members of P2PU are doing a lot of work, and the project is moving forward, with grant funding, incorporation, and several new staff positions. However, apart from contractual agreements within the nonprofit and basic standards for etiquette, participants have little or no accountability to one another. The P2PU governance model was said to be based on "*rough consensus*," after MIT professor David Clark's description of the Internet Engineering Task Force at the July 1992 IETF conference: "*We reject: kings, presidents and voting. We believe in: rough consensus and running code.*"† Pursuant to this, it would be helpful if a concise discussion of the current consensus norms and "best practices" for organizers and participants could be made available. The current *Course Design Handbook* provides one starting point, but it would represent a better consensus if it was improved by capturing insights from successful and less successful courses.‡ This sort of resource would be particularly useful for newcomers and to people who cannot regularly attend the community telephone calls.

Peers provide feedback that wouldn't be there otherwise: Dealing with problems in a respectful way Discussions about P2PU happen in the community mailing list and other places mentioned above. At the time when we were doing this research,

*https://wiki.mozilla.org/Drumbeat/SoW-engagement-ladder

†http://www.wired.com/wired/archive/3.10/ietf_pr.html

‡http://wiki.p2pu.org/w/page/27905271/Course-Design-Handbook

bug reports and feature requests related to the software were supposed to go into an issue tracker hosted on Lighthouse.* Discussions about P2PU also happen in courses, where they may be viewed as distractions from the content of the course. Even within the mailing list, it can be difficult to keep track of the full range of ideas circulating at any given time. And, apart from development work, it can often be hard to tell what's happening around P2PU. Presumably participants who have identified critical and unsolvable problems simply leave. The Q&A tracker† and mailing list both provide ways to build factual knowledge, but seem less effective for building *strategic* knowledge. Broadly, we would like to have had more clarity about how to contribute to the process of shaping P2PU.

Learning is distributed and nonlinear: Design considerations Facilitators are supposed to choose and assemble suitable learning resources for their courses (blogs, OER, etc.), in which everyone is supposed to learn something. In our experience, this is essentially what happened, but it is hard to measure when, whether, and how much knowledge was gained. Dealing effectively with complexity is hard, and more attention to modeling the learning process could help. However, it is not clear how or whether the data model used by P2PU's current platform – based on shared activity streams drawn from around the web – relates in a significant way to *learning*. It seems that P2PU would have to work hard in order to be able to use anything but "participation" as a proxy measure for learning. In terms of broader issues of quality control, one suggestion would be for P2PU core members – including staff – to start using the P2PU platform to organize their activities in the open, sharing what they are learning in the process of working on the site.

Realize the dream if you can, then wake up: High level roadmap At one time, the high-level vision at P2PU was arguably a Declaration of Independence from Formal Education.‡ But arguably each participant has their own vision, and their aims are often much less radical.§ When we were building this analysis, P2PU had recently had its first board meeting, but, so far, documentation about the organization's vision and roadmap have not been presented to or affirmed by the user community – nor has the user community presented any stipulations to the organization. P2PU has made considerable progress (e.g. in the form of successful grant applications), but without more transparency about these efforts, the ability of non-core members to learn from organizational successes is limited. In order to maximize the possibilities for others to contribute, P2PU should work on a public roadmap that leads from now up to the point where the vision is achieved. Both vision and roadmap should be revised as needed.

*http://p2pu.lighthouseapp.com/dashboard

†http://help.p2pu.org/kb

‡http://www.youtube.com/watch?v=t8wxUbU1W_0#t=12m11s

§http://www.youtube.com/watch?v=t8wxUbU1W_0#t=13m12s

3.3 Paragogical praxis

The study at P2PU helped develop a notion of "principled paragogy" – an essentially normative theory. These principles can be re-evaluated to set the stage for "emergent paragogy" – a descriptive theory. The paragogy principles point to five specific kinds of change that actions within a learning context can accomplish (Corneli & Mikroyannidis, 2011). These five points express the paragogical principles in a more uniform way.

1. **Changing the nature of the space** : Δ *context*
2. **Changing what I know about myself** : Δ *metalearning*
3. **Changing my perspective** : Δ *feedback*
4. **Changing content or connectivity** : Δ *distributivity*
5. **Changing objectives** : Δ *achievement*

If change is a "*quintessential element*" in learning (Burger & Starbird, 2012), it makes sense to give it a central role in learning design. The question "what changed?" provides a unified lens through which to examine and evaluate problem solving heuristics, like Pólya's (1945), as well as more open-ended problem solving processes, like those outlined by Schoenfeld [§1.3, p. 9]. Chapter 7 will use these five dimensions of change to understand user requirements for new problem solving support in PlanetMath/Planetary. Paragogy is a theory of peer learning in which the learner is not a separate species. With this in mind, the five paragogy principles can also be translated into practical suggestions on how to be an effective paragogue (Corneli, 2012). The following points outline a method for studying how the five factors of change mentioned above *themselves* change. With care, this engine can be run in reverse – not just to study change, but to bring it about.

1. **Develop empirical studies and a critical apparatus** : ∇ *context*
2. **Find companions for the journey** : ∇ *metalearning*
3. **Work with real users** : ∇ *feedback*
4. **Study and build nonlinear interfaces** : ∇ *distributivity*
5. **Limit philosophizing** : ∇ *achievement*

Chapter 5 describes the connections between these five dimensions and the design pattern methodology that is applied in Chapter 9. Together, the two levels of change are similar to what Gregory Bateson calls *proto-learning* and *deutero-learning*, or, more plainly, Learning I and II (Bateson, 1972, pp. 174, 292–306). Bateson's perspective on learning is discussed in Chapter 4, where it is used to illustrate the connections and differences between paragogy and other contemporary learning theories.

3.4 Summary

This chapter presents paragogy, a new theory of peer produced peer learning. Paragogy is different from earlier theories of learning, in the sense that broader contextual change is considered along with change in the persons who are learning.

Although paragogy initially developed in the mode of constructive critique, it is not tied to a project of "*fixing*" education; *cf.* (Kernohan, 2013). Indeed, paragogy is not presented primarily as an educational approach, but as a description of the way learning works in peer production communities. Paragogy has been amplified and extended within the Peeragogy project (Rheingold et al., 2014), which has helped make it useful to educators (Purser, Towndrow, & Aranguiz, 2013; Baker & Surry, 2013), and others (Corneli et al., 2014).

In order to give a foundation to the key ideas in paragogy, the presentation in this chapter has drawn on course design, participant observation, and on subsequent papers that explored the relevance of paragogy for system design and research (Corneli & Mikroyannidis, 2011; Corneli, 2012).

4 Literature Review

While there is no significant prior literature on "peer produced peer learning," the paragogy framework suggests a way to organize and survey prior work in adjacent fields.

4.1 Prelude: Learning and adaptation

This section will explore *learning*, focusing on Gregory Bateson's (1972) perspective, which was introduced briefly in the previous chapter [§3.3, p. 41], connecting it to the notion of *adaptation* that appears frequently in anthropological literature. The recent work of Andreas Meiszner (2010), who studied learning in the setting of free software and open courses, helps to show how these ideas relate to practical application and to contemporary learning theories. In the later sections of this chapter, this material will be useful in analyzing the literature related to peer produced peer learning in mathematics.

Bateson's hierarchy *Learning I*, *Learning II*, *Learning III*, etc., begins with *Zero Learning*, which denotes no change in the subject, only a stimulus and a reaction that is already "*soldered in*" (Bateson, 1972, p. 288). As with paragogy, Bateson understands learning to be fundamentally related to change, but in his model, he imposes the requirement that the environment should *not* change while learning is happening. His assertion is that "[t]*he notion of repeatable context is a necessary premise for any theory which defines 'learning' as change*" (Bateson, 1972, p. 296). Bateson does allow a hedge, which is that the context is only "*somehow*" or "*theoretically*" the same. The problem is that, here, Bateson is understanding context as "*a metamessage which classifies the elementary signal.*" Rather than talking about a repeatable context, it would be clearer to simply say that the stable classification of signals is what is important. Even in the case where a

bell sometimes means dinner and sometimes means no dinner, there is something to be learned, which is that the bell is capricious. All classification systems must "*fit innumerable, more or less similar cases – which means, strictly speaking, never equal*" (Nietzsche, 1873). Indeed, in some sense, what one is learning is precisely the classification system, with whatever built-in uncertainty pertains to it. This is compatible with Bateson's more formal definition: "*Learning I is change in the specificity of response by correction of errors of choice within a set of alternatives*" (Bateson, 1972, p. 298). The definition of Learning II, naturally enough, builds on the definition of Learning I:

> *Learning II is a change in the process of Learning I, e.g. a corrective change in the set of alternatives from which choice is made, or* [...] *a change in how the sequence of experience is punctuated.* (Bateson, 1972, p. 298)

Getting better at solving problems of a given type over time is an associated phenomenon, which related to the notion of *transfer of learning* whereby a problem in a new context is treated as if it were the same as a problem encountered in an earlier training context. Again, context change is perfectly compatible with learning at this level.

As an important class of examples, day-to-day communication works largely due to the fact that "*the stream of events is commonly punctuated into contexts of learning by a tacit agreement between the persons regarding the nature of their relationship*" (Bateson, 1972, p. 304). The pattern by which we should define Learning III and so forth is now relatively clear. Since "*what is learned in Learning II is a way of punctuating events*" (Bateson, 1972, p. 305) then what is learned in Learning III is a way of punctuating the punctuation of events – and so on. In other words, Learning II is about creating the boundary conditions in which a given pattern of control will be exercised, subject to the standard caveat that this control is partial. In a paragogical view, punctuation of this sort could also be called *context creation* – although often more is involved in context creation than just bracketing.

Bateson introduces several additional ideas that expand on this hierarchy, and which will be useful in what follows. The first is the distinction between analog and digital signals. Often the two are combined, as in language learning (Iverson & Thelen, 1999); indeed, "*there is a continuous gradation from the ostensive through the iconic to the purely digital*" (Bateson, 1972, p. 296).

For example:

> [A]*n English-speaking mathematician confronted with a paper by a Japanese colleague* [...] *gazes uncomprehendingly at the Japanese ideographs, but he is able to partly understand the Cartesian graphs in the Japanese publication.* (Bateson, 1972, p. 378)

Digital signals generally have conventional interpretations (*e.g.* the bell), whereas analog signals are more direct (*e.g.* the dinner), and need not stand for anything else. Clearly any conventions must be learned in order to be used. "*Verbal language*," in particular, is "*almost (but not quite) purely digital*" – and furthermore, "*primarily oriented toward things*," rather than "*the patterns and contingencies of relationship*" (Bateson, 1972, pp. 378, 377).*

Here, *relationship* especially means a context for communication, which as we've seen, means a context for learning, and, more broadly, a context in which cybernetic control is exercised. Bateson shows through many examples that humans and other mammals typically communicate about their most meaningful relationships with one another in nonverbal – *i.e.*, analog – ways. He introduces the term *μ functions* to describe communication acts, whether digital or analog, spoken or unspoken, which are ways of "voicing" relationship.

> *We shall say that "A dominates B" if A and B show by their behavior that they see their relationship as characterized by sequences of the type $a_1 b_1 a_2$ where a_1 is seen (by A and B) as a signal defining conditions of instrumental reward or punishment; b_1 as a signal or act obeying these conditions; and a_2 as a signal reinforcing b_1. Similarly we shall say that "A is dependant on B" if their relationship is characterized by sequences $a_1 b_1 a_2$, where a_1 is seen as a signal of weakness; b_1 as a helping act; and a_2 as an acknowledgement of b_1.* (Bateson, 1972, p. 305)

Some examples may help clarify these concepts. Wittgenstein's builders operate at the level of Learning I and execute a digital program – "*so that A can shout 'red slab!', and B will pass slabs of a certain colour*" (Bloor, 1983, p. 22). Given the thin description, we don't know for sure that *A* dominates *B*, which is certainly what it sounds like, or is perhaps dependent on *B*. In any case, the language of the builders is entirely about things, and the relationship is "voiced" through the embodied pattern of compliant acts.

In a real-world example, Richard Stallman solved a problem with printer jams at the MIT AI lab by installing custom code "*to notify every user with a waiting print job that the printer was jammed.*"

> *"If you got that message, you couldn't assume somebody else would fix it," says Stallman, recalling the logic. "You had to go to the printer. A minute or two after the printer got in trouble, the two or three people who got messages arrive to fix the machine. Of those two or three people, one of them, at least, would usually know how to fix the problem."* (Williams & Stallman, 2010, p. 3)

*Note that Geertz does not precisely say that a *symbol* is digital, but his list of exemplars – "*any object, act, event, quality, or relation which serves as a vehicle for conception*" – is comprised of discrete terms, rather than processes; in particular, a *relation* rather than a *relationship*. *Cf.* [§1.4, p. 9].

This message had the μ function of expressing a relationship between particular individuals and a printer, but it also served as a collective expression of the mutual interdependence of the members of the lab as a whole. This episode was an example of Learning II, enacted in such a way as to change the punctuation of experiences for the group. Furthermore, it served as a motivating example for an episode of Learning III, leading to the creation of the GNU project – based on the observation that custom messaging was only possible because the source code that controlled the printer's was accessible and could be changed.

It is now possible to make connections from this theoretical view to some of the standard learning theories. Here, I will largely follow in the footsteps of Andreas Meiszner (2010), whose PhD thesis asks whether the kind of learning that happens in free software communities can be applied to build open courses. Chapter 2 was framed around a similar motivation: "Can we make learning mathematics more like learning in free software?"

Meiszner draws on the description of self-directed learning from Gerald Fischer and Eric Scharff (1998), who write about the potential of "*Economies of Educational Knowledge*" that can be used by self-directed learners. According to Fischer and Scharff:

1. **Learning requires environments in which learners can be active designers and contributors.**
2. **Learning is highly tuned to the situation in which it takes place.**
3. **Learning is knowledge-dependent.**
4. **Learning needs to account for distributed cognition.**
5. **Learning is affected as much by motivational issues as by cognitive issues.**

These points bear a manifest similarity to the paragogy principles [§3.1, p. 36], however Fischer and Scharff retain a basically provisionist orientation. Their primary interest is in "*support for*" self-directed learners who act "*primarily as designers of some artifacts*." This is, indeed, the typical case in peer production, and the basic form in *constructionism* (Papert & Harel, 1991). However, as there is little emphasis on co-creating the learning context, this is a kind of "proto-paragogy."*

By and large, Meiszner finds the above-listed characteristics "*well presented within FLOSS*." He documents further specific features of the way learning works in free software communities, for instance, he notes the "*fragmentation and self-selection of learning materials*." He extracts sensible-sounding recommendations for designers of open courses, for example: "*Enable learners to engage* [with] *activities that are not restricted to*

*Fischer and Scharff's primary sources are Lauren Resnick (1989) for the first three points, who grounds them in the cognitive science literature; the contemporary work of Mitchell Resnick (1996) on distributed constructionism for the fourth point; and Mihály Csíkszentmihályi (1990) for the fifth.

an artificial university setting" and "*Implement peer support mechanisms - a large support network provided voluntarily by peers in a collaborative manner nearly 24/7.*" The problem then becomes, who will implement these proposals?

Connectivism, introduced by George Siemens, emphasizes the role that "*self-organizing networks*" and "*actionable knowledge*" play for learning in the "*Digital Age*" (Siemens, 2005). Massive Open Online Courses (MOOCs) designed with a connectivist orientation have many features matching Meiszner's recommendations. However, there seem to be some important differences.

> [O]*ne of the cornerstones of learning in FLOSS, is to 'enable re-experience', to 'learn from what others did',* [...] *Most of the discourse, debates or discussions of the CCK08 course are very close to the FLOSS case, a significant difference however is that those activities do not culminate within a 'solution', 'product', 'guide', 'report' or other type of 'concrete outcome' that would allow future cohorts to take this outcome as a starting point.* (Meiszner, 2010)

As the *Connectivism and Connective Knowledge* course was re-run a few years later as CCK11, this assertion must be questioned.* Nevertheless, the emphasis for most participants in connectivist MOOCs has been on "*dialogical*" features (Ravenscroft, 2011) – crafting an experience rather than a product. Without a doubt, such conversations are peer produced – but, still, without any aspersions, this is the kind of learning that takes place on "*Facebook, Twitter, G+, blogs, and other media*" (Purser et al., 2013), rather than the kind of learning that takes place in free software projects. In framing the central tenets of connectivism, George Siemens remarks that "*The pipe is more important than the content within the pipe*" (Siemens, 2005). The broad success of technologies for communication like StackExchange in learning applications notwithstanding, the paragogue would rather side with René Magritte here – "*Ceci n'est pas une pipe.*" Rather, after Benkler,

> *There is simply the question of governance in the relations among users of a class of software platforms that have certain degrees of freedom in their design, resulting in a variety of social affordances, and therefore facilitating a variety of social and economic interactions.* (Benkler, 2006b)

In this respect, paragogy is closer to *constructivism*, inasmuch as the latter, at least in its "radical" form, places an emphasis on *adaptation* rather than on knowledge transfer. As per Ernst von Glasersfeld, "[t]*o be adapted* [...] *means no more and no less than to be viable*" (von Glasersfeld, 1996). However, while von Glasersfeld intends for radical constructivism to be compatible with "*an ontological 'reality'*," he asserts that his framework "*denies the human experiencer the possibility of acquiring a true representation of*

*http://cck11.mooc.ca/

it" (von Glasersfeld, 1996). Bateson, for his part, would reject the separation of mind from body that this entails (Bateson, 1972, p. 470). Von Glasersfeld's focus on the construction of conceptual schemes and models leaves out broader contextual and relational features that are present, for example, in Vayda and McCay's perspectives on environmental change and group-level action and selection, which consider the adaptivity of the entire "*process of response*" [§3.1, p. 37].

Paragogy not only "*eschews pipeline models of transmitting knowledge*" (Papert & Harel, 1991) but treats communication as an assemblage with emergent properties, co-constituted by its participants, and subject to the vagaries of existence in the real world. In this, respect, we find paragogy at once in both its (em)*phatic* and *productive* senses.* Following Malinowski (1936), Julia Elyachar explains her concept of *phatic labor*:

> *Malinowski shows how language such as gossip and chatting can be a means of establishing ties for their own sake, rather than for the purpose of conveying any information in particular.* [...] *I argue that this labor produces communicative channels that can potentially transmit not only language but also all kinds of semiotic meaning and economic value.* (Elyachar, 2010)

The synthesis of a channel relies on the key feature that makes context creation different from the mere punctuation of experience, and different from standard models of communication: "*There is always more in the effect than there was in the cause*" (Harman, 2008). But these emergent effects must be *channeled*, or else they dissipate (Deacon, 2014). In short: paragogy is concerned with generating and voicing the μ functions that sustain relationships in which learning can take place, and through which meaning can be made. This process is "*morphogenetic*," not "*hylomorphic*" (Ingold, 2013, pp. 21–22) – but moreover it is associated with problems which possess an objective reality, as when "*a population of interacting physical entities, such as the molecules in a thin layer of soap* [...] *adopt a form which minimizes free energy*" (DeLanda, 1998). In the case of PlanetMath, with reference to the framework from Chapter 3,

1. **The co-created context is a mathematics website,**
2. **in which learning is part of a social knowledge building process,**
3. **carried by mediated peer feedback,**
4. **which unfolds in an architecture devised to support collaboration;**
5. **wherein the "dream" is one of shared community.**

Bringing together and contextualizing accounts from key figures associated with these themes, the chapter arrives at a picture of mathematical sociality that can be used to reframe the research question as an array of related research problems (Chapter 5).

* *Cf.* [§1.6, p. 14, Footnote ∗].

4.2 Mathematical thinking

How do people understand mathematics? This is the key issue in William Thurston's paper, "On Proof and Progress in Mathematics". Thurston had this admonishment for the mathematics community:

> *We need to focus far more energy on understanding and explaining the basic mental infrastructure of mathematics – with consequently less energy on the most recent results.* (Thurston, 1994)

human language
sensory perception
logic and deduction
intuition, association, and metaphor
stimulus/response
process and time
socialization

Table 3: *Criteria from William Thurston*

Note the emphasis here is on the *human experience* of mathematics. Thurston particularly emphasizes the importance of a social dimension for building a successful mathematics research programme. He begins with a point drawn from his own experience and very similar to Eric Raymond's insight that "*some very successful projects become 'category killers'*" (Raymond, 1998). Rather than clearing out mathematical topics on one's own, he observes that the long run it is much better to involve other people.

Thurston points out that people are able to make use of a wide, embodied channel to get involved. The way people *think* about mathematics (Table 3) is vastly different from what we see in the relatively dry and disembodied channel that typifies mathematics *papers*. Thurston describes mathematics as "*the theory of formal patterns*."* Nevertheless, he argues that

> [R]*eliability does not primarily come from mathematicians formally checking formal arguments; it comes from mathematicians thinking carefully and critically about mathematical ideas.* (Thurston, 1994)

The heart of mathematics *Problem solving* is considered by Paul Halmos to be "*the heart of mathematics*" (Halmos, 1980). Together with Thurston's definition, this speaks to a dual role for problem solving activities: they cross back and forth between the informal, experiential, "human" ways of thinking about a problem, and more "formal" static results (solution, proof, often "correct", etc.). "[T]*he mathematician's main reason for existence is to solve problems*" (Halmos, 1980). Adding problem solving interactions is a key "ergonomic" step in PlanetMath/Planetary, since it brings it into closer alignment

*He remarks, "*to discuss this would be a whole essay in itself.*" Thurston may be thinking here of formal structural organizations, not only in terms of logic; *cf.* (Thompson, 1917), (Alexander, 1964, p. 134). Thurston's former student Benson Farb indicates: "He changed our idea of what it means to 'encounter' and 'interact with' a geometric object. The geometry that came before almost looks like pure symbol pushing in comparison." HTTP://BLOGS.SCIENTIFICAMERICAN.COM/OBSERVATIONS/2012/08/23/THE-MATHEMATICAL-LEGACY-OF-WILLIAM-THURSTON-1946-2012/

with what people *do* when they do mathematics. Michael Atiyah argues that the "*core*" of mathematics is has as least much to do with theory building as it has to do with problem solving:

> [W]*hat is the good of a theory if it does not solve problems and what is the good of an infinite collection of disjoint problems, however interesting each individual one may be.* (Atiyah, 1974)

Timothy Gowers emphasizes that both problem solving and theory building can achieve the important end of organizing and communicating mathematical thought (Gowers, 2000). It would seem that communication lies near the heart of mathematics – and if that is so, then surely the choice of media for mathematical communication matters.

Studying mathematics online Khan Academy was discussed briefly in Chapter 2. If we look beyond the video format and the associated flipped-classroom pedagogy (Bergmann & Sams, 2012; Khan, 2012) we can say a bit more. Remarking on an inherently collaborative online mathematics learning environment, the Virtual Math Teams project, Gerry Stahl suggests that people studying online collaboration are in an uniquely good position, since they have access to the same information that participants in these environments have (Stahl, 2010). This is not even the case for persons researching *offline* interaction, who typically need to apply instruments like think-aloud protocols to build an explicit trace of what people are thinking (Nielsen, Clemmensen, & Yssing, 2002). (Of course, in practice, the users of an online system may have scratch work or side-conversations that are in no way indexed by the system.) One exemplary piece of recent research in this genre is a project on collaborative online peer supported proof-generation exercise that was conducted by Alison Pease and Ursula Martin. "Seventy four minutes of mathematics" (Pease & Martin, 2012) analyzes the activities that took place during *Mini-Polymath III*, a project in which Terrence Tao invited students to collaborate on his blog to solve an old Mathematical Olympiad problem. Having access to a trace of the entire conversation allowed Pease and Martin to categorize contributed comments as *concepts*, *examples*, *conjectures*, or *proof-related* – a framework inspired by Imre Lakatos's (1976) theory of informal mathematics.

Questions of procedure The Polymath project, which served as the inspiration for the Mini-Polymath series, aimed to answer the question "*is massively collaborative mathematics possible?*"* This project was convened by Timothy Gowers, who developed 15 rules to address the "*questions of procedure*" that he anticipated would arise.†

*http://gowers.wordpress.com/2009/01/27/is-massively-collaborative-mathematics-possible/

†http://gowers.wordpress.com/2009/02/01/questions-of-procedure/

In particular, the rules emphasize that a good contribution represents a "*quantum of progress*." Gowers progressively and carefully explicated the imperative to share comments that are "*not fully thought out.*" As Ursula Martin had commented ten years earlier with regard to the emerging field of *experimental mathematics*, that the growth of experimentation requires the "*development of community standards as to experimental methodology*," emphasizing not only technical matters, but also "*the early sharing of insights*" (Martin, 1999). Community standards are essentially synonymous to community norms, like those we would have liked to see more clearly elucidated at P2PU (Chapter 3 [§3.2, p. 39]). Community standards are part of any technology for shared communication; without them, communication is more difficult, or impossible; *cf.* Bateson on the tacit agreements that create contexts for learning [§4.1, p. 44].

Gowers's rules provide one starting point for community standards in *social* experiments in mathematics. These rules suggest a fairly precise understanding of mathematical socialization. It is not simply a matter of chatting things over, rather, it is a slow process of big ideas, and a faster intermediate process of breaking these ideas down into smaller parts. This recovers the *modularity* and *granularity* aspects the Commons-Based Peer Production model. Ultimately, participation in the Polymath project was not as "massive" as had been hoped (Gowers & Nielsen, 2009). This points to a potential expanded role for *integration*: not only to connect ideas into finished products, but also to connect the workflows of more persons. At the same time, the question about massive collaboration in research needs to be made more precise. Reasonably large scale collaboration in research mathematics had been going on much earlier, after a fashion: consider Mersenne's 17th century system (Bartle, 1995); and for a review focusing on Bourbaki, see Leo Corry on "Writing the Ultimate Mathematical Textbook" (Corry, 2009).

Making mathematical meanings There is a correlate to the quantum of progress in research mathematics to be found in a paper by Constant Leung about the process of learning elementary mathematics (Leung, 2005). Leung mentions three key points about learning mathematical language, and *mathematical vocabulary* in particular. Leung points out that when you learn mathematics, you have to learn both "*formal*" and "*semantic*" features of the concepts involved*; you have to "*think through*" the concepts involved; and that learning it is "*incremental*," in that meanings expand and develop by building on one another. This tells us what mathematical vocabulary does. Introducing new terminology is a way to represent new ideas – a quantum of progress in a learning or discovery process. It would not be entirely unreasonable to take learning vocabulary as a model for learning mathematics.

*Thus, for instance, a square is "*a figure with 4 straight equal sides forming 4 right angles*" (*formal/core*), and "*it has four lines of symmetry, and rotational symmetry of order 4*" (*semantic/non-core*).

> [W]*ords don't just represent what we claim to know already, they also allow us to make observations and to formulate novel meanings within a negotiated range of acceptable/accepted possibilities or limits.*" (Leung, 2005)

Mathematical semiotics Michael Halliday's view that a language contains terminology *and* a theory of human experience *and* an enactment of interpersonal relationships (Halliday, 2006, p. 50) is consistent with Leung's view that learning words is actually a matter of learning and enacting process, and consonant with Thurston's perspectives on mathematical experience. Halliday makes it clear that learning scientific or mathematical language is not just a matter of learning new vocabulary words. Writing about students learning science, Halliday said that the difficulty is not just with "*technical terms*" but with the "*the total patterns of the discourse*" (Halliday, 2006, pp. 200–201). The salience of these comments is amplified when we consider Halliday's language-based theory of learning, which says that, for humans, learning how to relate to others, and indeed learning how to think, are intrinsically linguistic processes. To illustrate these claims, Halliday presents a detailed study of the development of thought through 21 distinct features of childhood language development (Halliday, 1993). In this work, all human learning activities are seen as "*semiotic*": the system of language is concomitant with processes of meaningful acts. Halliday's thinking about system and process is reminiscent of Louis Hjemslev's thesis:

> [F]*or every* ***process*** *there is a corresponding* ***system****, by which the process can be analyzed and described.* (Hjelmslev, 1961, p. 9), original emphasis

In this way, we move back and forth between words/symbols, system/language, and process/meaning.

Learning mathematics from examples (and by doing) But does a language-based theory adequately describe the way people learn mathematics? A paper by Xinming Zhu and Herbert A. Simon demonstrates the feasibility of an alternative learning-by-example approach (Zhu & Simon, 1987). In this case, the examples were spelled out in advance in textual form. The underlying model is simple, and best suited for rote learning (Waterman, 1975). This approach to learning from worked-out examples contrasts strong with the "*Moore method*" (Jones, 1977; Cohen, 1982), a variation on the practice known as "*discovery learning*" (Bruner, 1961). Here, the syllabus consists of a selected list of theorems to prove. Clearly, in order to prove non-trivial theorems, one needs "*a reasonable degree of sophistication concerning the logic of verification and concept formation*" (Strike, 1975). The Moore method generally relies on the rediscovery of a known system behind the revealed process. On the other hand, the Zhu/Simon model is an example of a system with only a minimal process: it provides a limited language that only

- If a problem seems familiar, try *reasoning by analogy.* If you solved a similar one in the past, and can adapt to the differences, you may be able to re-use that solution.
- If the problem still seems too hard, *divide it into several parts.* Every difference you recognize may suggest a separate subproblem to solve.
- If it seems unfamiliar, *change how you're describing it.* Find a different description that highlights more relevant information.
- If you get too many ideas, then *focus on a more specific example* – but if you don't get enough ideas, make the description more general.
- If a problem is too complex, *make a simpler version* of it. Solving a simpler instance may suggest how to solve the original problem.
- Asking "*what makes a problem hard?*", may suggest another approach – or a better way to spend your time.
- When your ideas seem inadequate, remember someone more expert at this, and *imagine what the expert would do.*
- Whenever you find yourself totally stuck, *stop what you're doing* now and let the rest of your mind find alternatives.
- The best way to solve a problem is to already *know how to solve it* – if you can manage to retrieve that knowledge.
- If none of these methods work, you can always *ask another person for help.*

Table 4: *Problem-solving heuristics suggested by Minsky, with new mnemonics*

makes it possible to reexpress the guiding text. When compared with Halliday's detailed developmental model of learning as a linguistic/semiotic/meaning making process, both of these models appear quite flawed.

A linguistic desert In a series of memos written for the One Laptop per Child (OLPC) project,* Marvin Minsky points out that young students are not given many mathematical words or ideas. "*It is hard to think about something until one learns enough terms to express the important ideas in that area*" (Minsky, 2008–2009).† However, Minsky's concern is not solely with increasing the rate of vocabulary acquisition, but with learning new ways to think and problem solve. He proposes several example heuristics (*i.e.*, ways of thinking about problems) that could be taught to students (Table 4). In the OLPC memos, he suggests that programming may be a better way to teach heuristic reasoning than mathematics. Elsewhere, Minsky emphasizes that to build real understanding, it is necessary to be able to "*represent something in several ways*" (Minsky, 2007, p. 6).

*HTTP://ONE.LAPTOP.ORG/

†*Cf.* Gerry Sussman: it's "*hard to learn what one cannot express*" [§3, p. 10].

4.3 Collaborative knowledge building

The model of learning from discovery and examples, where both are embedded in a social process, is similar to Marlene Scardamalia's (2002) conception of knowledge building, mentioned in Chapter 3. These ideas build upon earlier work that was developed jointly with Carl Bereiter (Bereiter, 1994; Bereiter & Scardamalia, 1993; Bereiter, Scardamalia, Cassells, & Hewitt, 1997). These works emphasize heuristic design guidelines such as: "*There is a way into the central knowledge space for all participants*" (Scardamalia, 2002).

Tacit preunderstanding
Personal attention (⋆)
Personal comprehension (⋆⋆)

(⋆) Personal attention
Public statements
Argumentation and rationale
Shared understanding
Collaborative knowledge
Cultural artifacts
(⋆⋆) Personal comprehension

Table 5: *Criteria from Gerry Stahl*

"A model of collaborative knowledge building" is a short paper by Gerry Stahl (2000), in which he talks about "*learning as a social process of knowledge building*." The aim of Stahl's paper is to add a cognitive dimension to the early, more informal, theories of knowledge building. In particular, the paper aims to mesh "*personal understanding*" (considered as a cycle) with "*social knowledge building*" (considered as another cycle) – see Figure 4.* Table 5 presents a minimal rendition of the diagram's main generative cycles and mutually-satisfying boundary conditions.

Stahl's model does an admirable job of organizing the key ideas of social knowledge building, while simultaneously offering a potentially descriptive theory. However, as a cognitive model, it remains fairly schematic.

Another candidate structuration In a recent paper by Sean Goggins and co-authors, Stahl's work on collaborative knowledge building inspires another graphical interpretation (Figure 5), where *context*, *people*, *artifacts*, and *interactions* are organized in *contextualized interaction* (Goggins, Mascaro, & Valetto, 2013). This model also considers people, artifacts, and a shared context. There is less of a focus on the particular *teloi* of belief, understanding, and comprehension. However, there is more of an emphasis on small-scale feedback loops, which are connected to a large-scale research cycle that aims to identify the social connections that are created within the groups being studied.

Modeling the process of learning "Modeling the process, not the product of learning" (Akhras & Self, 1999, pp. 3–28) was published around the same time as Stahl's short paper. Here, Fabio Akhras and John Self look for ways to model learning within a

*In this revision to Stahl's (2000) diagram, "*social knowledge building*" splits into "*small group interaction*" and "*commmunity knowledge building*", and what was formerly "*personal belief*" is now labeled "*personal attention*" (Stahl, 2013).

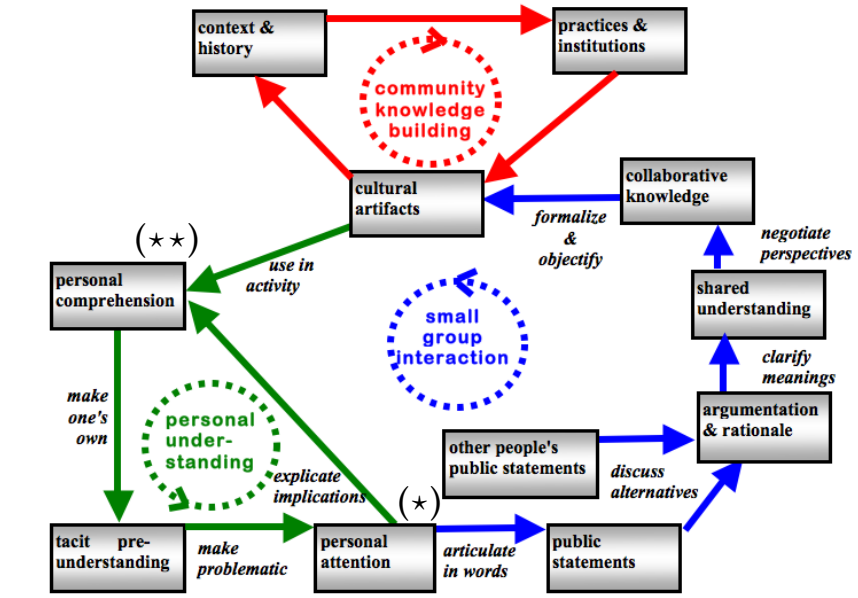


Figure 4: *Stahl's diagram of factors involved in collaborative knowledge building, Figure 8-4 from* Translating Euclid: Creating a human-centered mathematics, *© 2013 Gerry Stahl (used with permission; annotations* $(\star)$ *and* $(\star\star)$ *added to indicate the points of departure and arrival for the two main cycles)*

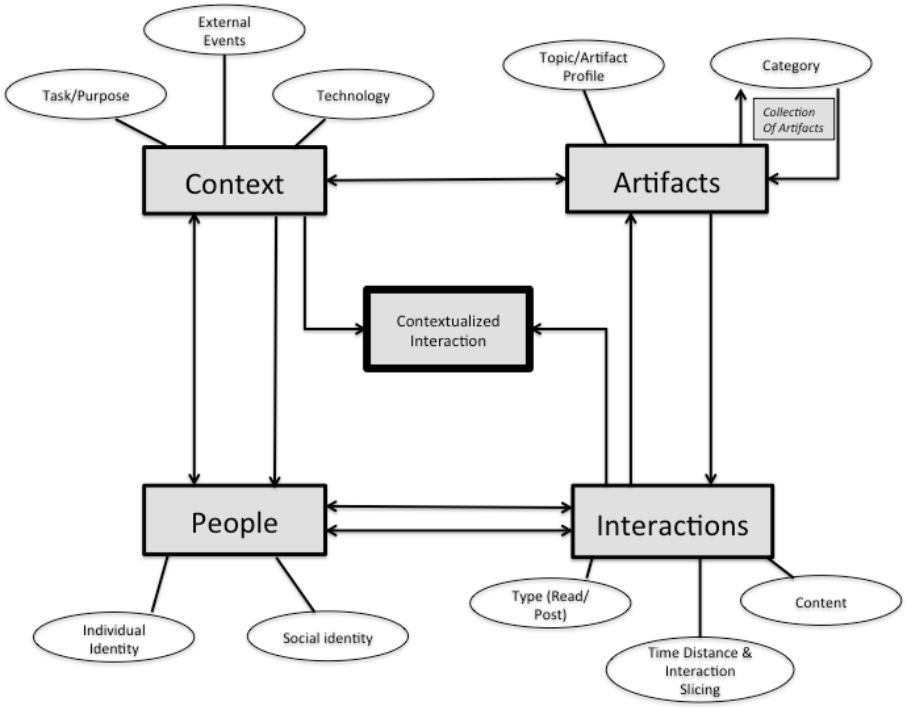


Figure 5: *Overview of the Group Informatics Model, Figure 1 from "Group informatics: A methodological approach and ontology for sociotechnical group research", © 2013 Sean Goggins (used with permission)*

"*constructivist mindset*." The classic approach to student modeling in intelligent tutoring systems seeks to capture domain knowledge in certain objective representations, which the student is then supposed to internalize – they cite Wenger (1987) as an example. According to Akhras and Self, knowledge is an inherently process-based phenomenon that "*cannot be objectively defined and statically represented.*" Akhras and Self therefor restrict themselves to modeling the "*context*," "*activity*," and "*temporal factors*" of learning. In a subsequent work, they go on to argue that "*if you want to take a constructivist approach the ITS framework will not help because of the different issues addressed*" (Akhras & Self, 2002). Commentators Michael Young et al. (2002) use Akhras and Self as a launch point to call for the consideration of the multidimensional nonlinear dynamics associated with learning interactions – a perspective that is quite sympathetic to the paragogical view.

The (multidimensional) Zone of Proximal Development As certain basic problems get easier to solve, people move on to more difficult ones. This principle is perhaps best studied within the field of child psychology, where it was formalized by Lev Vygotsky as the *Zone of Proximal Development*,

> *a sphere formed by the aggregate of vectors that pass through a 'point' of difficulty and that delineate a child's diverse possible areas of development.* (Zaretskii, 2009)

When the learner moves within this multi-dimensional "*zone*," he or she may overcome particular difficulties with the help of competent others. Once a given challenge can be satisfactorily met with help, it can often later be managed independently. Of course, we do not assume that social interactions, *per se*, will always have a positive effect on development or learning (Hogan & Tudge, 1999, p. 61). In particular, we cannot expect that a given peer will have the skills or sensitivities needed to help navigate and maintain the ZPD of another (*ibid.*, p. 57). The multidimensional Zone of Proximal Development provides a satisfying generalization of the basic form described by Stahl (see Figure 4 and Table 5, above). Both systems are reminiscent of the *microsphere* defined by Peter Sloterdijk, whose key feature is its bi-centered nature and whose basic paradigm is life in the womb (Sloterdijk, 2011, p. 540).

The emergence of mathematical meaning Cobb and Bauersfeld (1995) present an edited volume containing several detailed analyses of classroom-based teaching and learning practices. The focus is on examining what happens when the teacher shifts from a pedagogy revolving around teaching what is seen as the correct way to think about things, to pedagogy where the teacher is, or tries to be, genuinely interested in the way students think as well. For example, students are encouraged to come up with their own ways to add numbers like 56 and 67. In addition to the traditional strategy of lining up the numbers in columns and carrying digits, one might round down to 55 plus

65, see that this is the same as 60×2 and add back the 3 points from rounding at the end. Or one might work the sum as $(50 + 6) + (60 + 7)$. If not limitless, the varieties of mathematical experience are at least highly diverse, and it seems the teacher must quickly muster some patience to deal with the enthusiastic creativity of her pupils, while also remembering to encourage them to reflect on one another's ideas. This appears to be an effective way to teach mathematical creativity. However, one wonders how much interest the student answers really have for the teacher, especially after the curriculum has been run several times. Ilana Horn underscores that "[t]*he focus on student thinking requires a genuine curiosity about young people and their ideas*" (Horn, 2012). Relating recent pedagogical experiments in the JUMP Math project, John Mighton writes:

> *Research has shown that many elementary teachers* [...] *are mathphobic or have very rudimentary knowledge of math.* [...] *In following the online lesson plans* [from JUMP Math] *teachers learn the math as they teach. Many have become excited about their new understanding of the subject and have formed volunteer networks to support and mentor other teachers.* (Mighton, 2014)

These examples illustrate a view of mathematics education in which peer pedagogy and paragogy are potentially compatible, featuring ongoing, active, and creative learning for all involved. The emergence of mathematical meaning goes along with the formation of new practices, for example, effective practices of metacognition, as described by Alan Schoenfeld [§1.3, p. 9] – and may be accompanied by new social arrangements, as described by Mighton. Indeed, the social in Mead's sense of an unstable "*betwixt and between*" [§1.3, p. 8] tends to be well exemplified in learning; *cf.* (Deleuze, 1968, p. 204).

The following section emphasizes the role emergence can play in the co-creation of an environment that meets the proximally-developmental needs of the peer learner, whose not-just-(en)cyclical learning style has been traced above.

4.4 The ecological approach in learning design

"The Ecological Approach to the Design of E-Learning Environment: Purpose-based Capture and Use of Information About Learners" is a paper by Gordon McCalla that talks about how to use interaction data to help build a learning environment (McCalla, 2004). McCalla does not use the word "stigmergy" in this paper, but his view of mediated interaction matches that definition well, if we understand *stigmergy* to mean:

> [T]*he use of environmental conditions as instigators of action and the overall ability of the group to perform problem-solving activity that exceeds the knowledge and the computational scope of each individual member.* (Clark, 1998, p. 234)

A key part of McCalla's view is that the system adapts as its external environment

changes, where its "*external environment includes learners, teachers, the subject matter being learned, and the technology that implements the e-learning system.*" Such a perspective is in line with the broader *emergentist* approach in cognitive science (and robotics), in which systems are programmed directly by the environment, and can make use of environmental features for purposes of self-programming (Clark, 1998).

The idea in McCalla's model is to engineer systems that can capture information about users and individual use events. Data describing use events has elsewhere been given the convenient name "*paradata*," to distinguish it from "*relatively static metadata*" (National Science Digital Library, 2012).*

Gradual, localized, accumulation of information
Focus on information about the end user
Purpose-based use of data, in support of a pragmatic web

Table 6: *Criteria from Gordon McCalla*

These ideas coalesce into a scheme (Table 6) for capturing use data related to learning, by "*attaching*" a model of the learner to each object the learner uses. This model is considered to have "*characteristic*" and "*episodic*" aspects (*metadata* about the user, and *paradata* about the specific use event). The data is to be used by "*purpose-based*" data clustering and data mining algorithms.

Good contributions and good questions Q&A sites make reasonably transparent use of paradata (upvotes, downvotes, and accepted answers, as well as editing behavior) to award points and badges of markers of progress. They provide a concrete example of *marker-based stigmergy* in action, and a partial implementation of McCalla's vision. Since pragmatics is at issue, it's worth asking: what makes a good contribution on a Q&A site? The first basic finding is that good online contributions tend to mirror good offline contributions. Tausczik and Pennebaker (2011) showed that strong mathematicians (according to offline metrics) are more likely than other participants to ask highly rated questions on the mathematics research Q&A site MathOverflow. Anderson, Huttenlocher et al. (2012) examined MathOverflow's big sister, Stack Overflow, and found that questions that appear together with multiple different answers brings lasting value to the site. Another striking finding was that the events that take place within the first hour after a question is posted are reliable predictors of its subsequent interest up to a year later.

Reflecting on her experiences with students in an offline setting, Alison King (1999) captures the essential message: "*thought-provoking questions promote high-level discussion, which has been found to result in high-level learning.*" King divides questions into two categories, *comprehension questions* – which seem well-suited to dealing with factual

*The term "paradata" was previously used in an approximately similar manner in the context of survey-based research, see Safir et al. (2001).

material – and *connection* or *integration questions*, which are more suited to analyzing and integrating ideas. Combining these with *strategic* questions, peer learners can tackle ill-structured problems.

Still more mathematical meaning One can look for meaning throughout a wide range of actions and interactions, for example, solving a problem, asking a question, and giving or receiving a hint. These activities may have further subjective features, including "*processing level*," "*quality of self-explanation*," and "*interest in exploration*" (Hosein, 2009). Relational or contextual features of behavior are likely to be particularly significant. Chan, Lee, and van Aalst (2001), drawing on Scardamalia (2000), described "*Working at the cutting edge*," "*Collaborative effort*," and "*Identifying high points in the discourse*." There are many ways in which this sort of distributed collaboration can be recognized and persisted to add value to a resource; *cf.* Corneli and Mikroyannidis (2011).

Sticking things together "*Local solutions to local problems*" is the watchword for Elinor Ostrom's theory of commons governance (Ostrom, Burger, Field, Norgaard, & Policansky, 1999). Local features are also important in McCalla's proposal to implement educational systems using *localized* accumulation of information. Martin Nowak's work is tremendously useful because it points to a kind of glue – "*cooperation*" — that can be used to stick local regimes together (Nowak, 2006). Nowak's five rules for the evolution of cooperation are quite simple:

1. **Cooperate with agents to whom you are sufficiently related.**
2. **Cooperate with agents who help you.**
3. **Cooperate when it improves your reputation.**
4. **Cooperate with agents in your locale.**
5. **Cooperate with agents in a designated "group" that you belong to.**

A pragmatic web King's (1999) discourse patterns for collaborative inquiry provide a simple implementation of "*the gradual, localized, accumulation of information* [...] *in support of a 'pragmatic web'*." Following Geertz we might accept a more typical name for this sort of technology: "*culture*" (Geertz, 1973, p. 5).

The next section examines the practical features of culture formation in peer production communities, where knowledge is no less "*localized, embedded, and invested in practice*" than in the corresponding case of firms (Carlile, 2002) – but where "*integrating devices*" and "*representational capacity*" are perforce more explicitly artificial.

4.5 Architectures for collaboration

In Aaron Krowne's overview of the principles that informed the design of PlanetMath in its original incarnation, he states:

> *The basic, universal goals of digital libraries are to provide a logically organized, conveniently accessible, and (if possible) easily **actionable** collection of digitized **knowledge** in some field or fields for an audience of **learners**.*
>
> (Krowne, 2003B), original emphasis

As we've retraced his steps in the Planetary project, we've had the opportunity to re-examine the huge amount of often pioneering technical work that went into Noösphere. This work was nevertheless "of an era" – an era that can be understood more completely with reference to two other CBPP-related projects that started around the same time: Wikipedia/MediaWiki (Barrett, 2008) and Drupal.org/Drupal (Melançon et al., 2011). The key point, as we reflect back on the foundations of PlanetMath, is that there was a certain way of working, and indeed, a certain *theory of collaboration*, implicit the early architecture. The most noticeable tenets of this theory were associated with practices for "*homesteading the noosphere*" (Raymond, 1998) – namely object ownership and the corrections system. These features set PlanetMath apart from Wikipedia, and situate it closer to the typical practices associated with open source software.* PlanetMath, Wikipedia, and Drupal.org are all concrete examples of large postmodern organizations, built by many individuals who are asynchronously updating a shared database. The software systems behind the websites define and delimit many of the connections and boundaries between people, topics, and goals; *cf.* (Siemens, 2005), (Star & Griesemer, 1989).

Building large knowledge bases by mass collaboration The framework presented in Table 7 comes from Matt Richardson and Pedro Domingos (2003), who wrote about the particular challenges of building large knowledge bases through a process of mass collaboration. They also propose a specific approach to this problem, which cannot be given the same level of acclamation as their theoretical framework: it is hard to imagine inputting rules and facts as Horn clauses as a particularly popular pastime. The framework can be usefully applied to describe the process of "*building a digital library the commons-based peer production way*" (Krowne, 2003B), and, by extension, used as a lens for examining the social-technical-computational systems that *do* peer production.

Krowne's paper shows that a distributed approach can directly help to satisfy the criteria listed by Richardson and Domingos. For example, when compared with a centralized approach, commons-based peer production can help with *relevance*, insofar as:

* *Cf.* HTTP://DRUPAL.ORG/PROJECT/PROJECTAPPLICATIONS

Relevance. *In a distributed setting, ensuring that the knowledge contributed is relevant – and that volunteers' effort is productive – is a* [...] *significant problem.*

Quality. *Ensuring the quality of knowledge contributed by many different sources, when little is known about most of them, is likely to be very difficult.*

Scalability. *To achieve its full potential, a collective knowledge base must be able to assimilate the work of an arbitrarily large number of contributors, without the need for centralized human screening, coordination, or control becoming a bottleneck.*

Consistency. *As the knowledge base grows in size, maintaining consistency between knowledge entered by different contributors, or even by the same contributor at different times, becomes increasingly difficult.*

Motivation. *Following the example of open-source software, collective knowledge bases should allow user-developers to enter knowledge that is first of all relevant to solving their own problems.*

Table 7: *Criteria from Richardson and Domingos*

> [K]*nowledge is distributed unevenly and/or widely; a centralized effort can be inflexible; experts may be too busy with other work to commit entirely; and digital library builders themselves are not experts.* (Krowne, 2003B)

The architecture of theories To better understand the theory of collaboration underlying sites like PlanetMath, it is useful to enlist the help of Charles S. Peirce, who began his 1891 paper on "The architecture of theories" with a critique of those theories built from one idea, as if they were "*houses made of paper*" (Peirce, 1891). This would seem to be an especially important point for theorists of mediated collaboration to consider. A more recent critique focused on overly-constrained paradigms for the learning technology sector was advanced by Bernard Lisewski and Paul Joyce (2003), who argued that "*there are inherent dangers in them becoming too dominant a discourse*" or "*grand narrative*." They repudiate the uncritical use of theoretical systems. Table 8 shows how the framework proposed by Richardson and Domingos applies to the systems in operation at PlanetMath, Wikipedia, and Drupal.org. This table offers reassurance that each project has several important contributing ideas. Even so, a system needs to keep evolving in order *not* to become monolithic; *cf.* (Peirce, 1891).

Dictionary vs Encyclopedia Umberto Eco (1986) talks about written works that are organized along the model of the *Porphyry tree*, and works that are organized along the model of the *rhizome* or *labyrinth*, echoing (Deleuze & Guattari, 1980, pp. 5–7). Elsewhere, Deleuze emphasizes the associational aspects of cultural systems.

	PlanetMath	Wikipedia	Drupal.org
Relevance	Relevance depends on peer review, and irrelevant content may be deleted. The mechanisms that would ensure that relevant content *will* be added need improvement.	People contribute articles about what they're interested in; apart from this, rules like WP:WEIGHT come into play.	Anyone can upload projects, although "full projects", require one time approval. Changes to the core require more vetting.
Quality	Quality control is handled with corrections and the "orphaning" mechanism in case of nonresponsive authors. Some articles are world-writeable, as in the wiki model.	Automated tools for spam and vandalism detection combined with a system of editorial oversight, in which Jimmy Wales has last say.	In addition to bug reports and feature requests handled through the issue tracker, modules can make use of an automated patch testing system.
Scalability	Peer review is distributed. Links are handled automatically. Caching is deployed where relevant; in particular, interlinking features are kept up to date.	The database and other infrastructure is massively scaled. There are many bots that help with small tasks.	In theory, anyone can join. Earl Miles, NYC-Camp 2012 keynote: "*There are no insiders, except Dries; there are no outsiders, only resumes.*"
Consistency	Although automatic links and corrections can help with consistency, mainly PM relies on standards for proof and expository quality.	NPOV is the key rule, which works together with templates and other process tools to maintain community standards about style and content.	The project issue queues are the place to go when one module's changes breaks another's. The core of the project has considerable oversight in these matters.
Motivation	People are solving some of their learning, exposition, and social needs on the site by writing and reviewing articles and posting in the forums.	As of 2006, over 50% of the site had been written by less than 1% of the users; these days, paid editing is somewhat notorious.	Miles continued: "*To build a resume, find someone who needs help, and help them. Find something that needs doing, do it.*"

Table 8: *Criteria from Richardson and Domingos applied to describe three communities where mass collaboration happens (PlanetMath, Wikipedia, and Drupal.org)*

> [Hume] *gave the **association** of ideas its real meaning, making it a practice of cultural and **conventional** formations* [...] *rather than a theory of the human mind. Hence, the association of ideas exists for the sake of law, political economy, aesthetics, and so on. People ask, for example, whether it is enough to shoot an arrow at a site in order to become its owner, or whether one should touch the spot with one's own hand. This is a question about the correct association between a person and a thing, for the person to become the owner of the thing.* (Deleuze, 1953), original emphasis

Is PlanetMath best approached as an associationist dictionary or as a rhizomatic encyclopedia; *cf.* [§2.3, p. 30]? The specific example of *ownership* mentioned above is quite relevant in forming a prognosis. Following Raymond (1998), ownership after the Lockean property model was an important consideration in the design of PlanetMath/Noösphere (Krowne, 2003A, p. 5). As James Leach explains, in the Enlightenment tradition after Locke,

> *Human society is based on the ownership of property, as relationships between individuals **came into being** because of the appropriation of resources and the need to institutionalize that appropriation.* (Leach, 2007, p. 105), original emphasis

This way of thinking is based on the "*separation of spirit from matter, of appropriator from the substance appropriated*" (Leach, 2007, p. 105).* As described above [§4.1, pp. 47-48], this kind of thinking can truncate or distort important features of adaptivity. If "[t]*he knowledge needed to make land productive is part of a complex arrangement in social groups, and between them, whereby land and people become parts of one another*," and if people, more so than property, are "*the projects of other people*" then one "*distinguishes himself and gains authority through connection, not through exclusive control*" (Leach, 2007, pp. 111–112). At the very least there are some ideas here worth considering.

The next section treats other issues to do with relationship, community, and motivation that have relevance to paragogy.

4.6 Building online communities

Communities are often places where smaller communities form and dissolve again when their purpose has been served – recall the small naturally asynchronous groups of Goggins et al. (2013). One initial hypothesis that could help to understand communities is that they are most attractive when they work in a recursive fashion: *i.e.* healthy communities help their participants to develop sub-communities. In "Participation in an online mathematics community" Tausczik and Pennebaker (2012) use a framework adapted from Dholakia, Bagozzi, and Pearo (2004) to describe the reasons people participate in the research-level online mathematics community, MathOverflow (Table 9).

*Quoting (Pocock, 1985, p. 43).

Getting information
Giving information
Reputation building
Relationship development
Recreation
Self-discovery
Constructive Feedback

Table 9: *Criteria from Tausczik and Pennebaker*

Tausczik and Pennebaker examine each of these various motivations, using MathOverflow's built-in stigmergic mechanisms (score, downvoting, accepting answers, and comments) together with a textual analysis of comments conducted using the Linguistic Inquiry and Word Count (LIWC) methodology (Pennebaker, Francis, & Booth, 2001), which connects psychologically meaningful words to activities representative of each motivation.

Their findings report that "[t]*he strongest, most consistent evidence was found for the importance of reputation building in encouraging contributions*" however, there was only "*weak evidence that relationship building acted as an incentive to contribute*" and indeed that it "*may be another form of reputation building*" (Tausczik & Pennebaker, 2012). They emphasize the motivating features of concrete "*social benefits*," similar to Benkler's (2006a, 2002) notion of "*social-psychological rewards*." The model used on sites like MathOverflow seems to be an updated version of Eric Raymond's (1998) perspective: he suspected that "*the reputation-game gift culture is the globally optimal way to cooperate for generating* [...] *high-quality creative work*."

This would tend to go against the recursive hypothesis advanced above. Nevertheless, following Bateson, it is clear that relationships of one form or another are required in order for MathOverflow to support learning. The key point is that these relationships may be what Karin Knorr Cetina calls "*postsocial*" (Knorr Cetina, 1997): relationships not with people, but with mathematical objects and a distributed process of inquiry.

Personal motivations Daniel Pink (2010) believes that external rewards are often simply distractions, and proposes a theory of motivation based on "*autonomy*," "*mastery*," and "*purpose*." Thomas Malone (1981) advances a related framework for building motivating learning activities based on elements of "*fantasy*," "*challenge*," and "*curiosity*."

Building new online communities Even if all users are not motivated to participate in community-buliding activities, some are – and some may participate even without an explicit motivation. It is worth mentioning some points of alignment between Table 9 (particularly, reputation building and relationship development) and the advice from Resnick, Konstan, Chen, and Kraut (2012), who make these pragmatic suggestions to aspiring community builders:

> *– Drawing attention to external publicity and endorsements can raise expectations about future success.*
> *– In synchronous spaces that are not always active, a schedule of 'expected active times' coordinates visitors and can become a self-fulfilling expectation.*
>
> (Resnick et al., 2012)

Almost Wikipedia "Almost Wikipedia" was a thought-provoking talk given by Benjamin Mako Hill (2011A) at Harvard's Berkman Center, in which he says, in order to be as successful as Wikipedia, you should first make something people are familiar with – and second, make it easy for them to contribute.* In these terms, rebuilding PlanetMath as a collection of peer produced mathematical textbooks would make a good amount of sense. Sub-communities would also be readily recognizable (*e.g.*, "the authors of the Calculus textbook", *etc.*). In a recent paper, Hill and Monrey-Hernández (2012) point out a related finding: "*attributes of works associated with increased generativity are associated with decreased originality, and vice versa.*" Something that is easy to understand – and is, in that sense, perhaps not so novel – will be relatively easy to build upon.

4.7 Other related frameworks

Gregory Bateson explained the existence of a class of parallels arising in "*communicational and organizational process*" by reference to the "*the mysterious and polymorphic relation between context and content*" (Bateson, 1972, p. 163). There are many frameworks in the broader space of learning, communication, and organization theory that bear some analogy to the paragogy framework. These include Bateson's own criteria of mind (Bateson, 1980, p. 92 *et seq.*), and the functional framework of social movement theorists in the tradition of Luther Gerlach (Palmer, 2007). In short, if the paragogy principles are a rediscovery, they appear to be a very common one. As Harman (2008) remarks, "*anyone who looks seriously for fourfolds will start to find them everywhere*," and the same seems to go for 4 + 1 or 3 + *n*; *cf.* Genosko (2002, pp. 217–226).

The analysis that follows will draw upon some of these approximately-parallel frameworks, including: Pease and Martin's (2012) interpretation of Imre Lakatos's (1976) categories of mathematical activities, which is discussed in Chapter 7; Marshall McLuhan's tetrad of media effects (McLuhan & Powers, 1989), which is used to examine new features of Planetary in Chapter 8; and Stafford Beer's criteria for viable systems (Beer, 1981), which helps to frame discussions around projects in Chapter 9. In introducing the cybernetic notion of culture that was adopted in Chapter 1 [§1.4, p. 9], Geertz explained his motivations – which continue to apply here:

> [W]*e need to replace the "stratigraphic" conception of the relations between the various aspects of human existence with a synthetic one; that is, one in which biological, psychological, sociological, and cultural factors can be treated as variables within unitary systems of analysis.* [...] *It is a matter of integrating different types of theories and concepts in such a way that one can formulate meaningful propositions embodying findings now sequestered in separate fields of study.* (Geertz, 1973, p. 44)

*HTTP://CYBER.LAW.HARVARD.EDU/EVENTS/LUNCHEON/2011/10/MAKOHILL

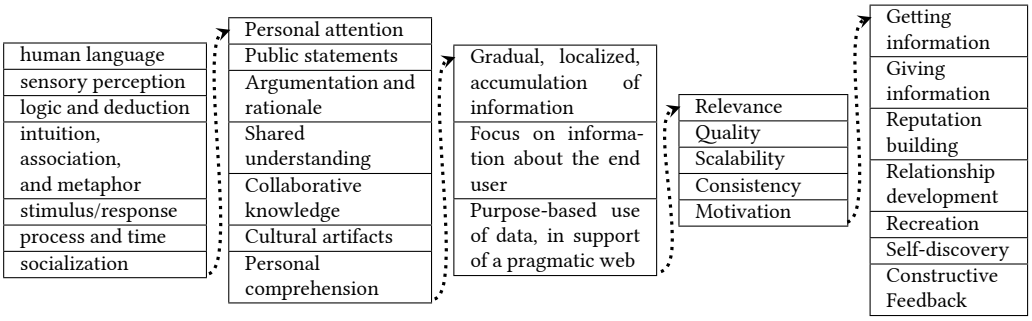


Figure 6: *Links between the key dimensions coming from the literature*

4.8 Summary

This chapter has contextualized the key pieces of research literature that are relevant to peer learning and peer production in mathematics. The framework provided by paragogy allowed a wide net to be cast, and provides an initial set of interlinked dimensions to consider when building a software model or further developing a paragogical approach in mathematics.

Arguing against any overly simple *narrative*, the aim has been to give paragogy a useful *form*. One possible instantiation is presented in Figure 6, which pulls together the main thematic ideas from the central figures discussed above, and shows that they fit remarkably well in a hierarchy. Socialization is important to Thurston: how does it work? This is explained by Stahl. And Stahl's critical personal comprehension? Perhaps this is built in a ecological way, as with the systems considered by McCalla. These rely on the idea of a pragmatic web, which could be described in terms used by Richardson and Domingos. Finally, the system as a whole is fueled by motivation, which has been examined in detail by Tausczik and Pennebaker. Whereas Maslow (1943) would presumably say that mathematics is pursued by generally healthy individuals as part of their efforts at self-actualization, this figure provides a more refined view. The relevant behaviors are inherently social – and the development of mathematics takes place across individuals. What we typically think of as the life of the mind may simply be "*the cutting edge of the life process itself*" (Ingold, 2000, p. 19).

Notice that the picture does not include the word "mathematics." It is possible to be more specific by thinking about how this scheme works in the mathematics case. After Halliday, understanding mathematics as a semiotic domain suggests that the core processes are not simply concerned with abstract symbol manipulation, but with making meaningful experiences. And after Bateson, it's clear that this requires the voicing of μ functions in which meanings can subsist. What is at stake in taking this forward is the matter of "*becoming relational*" (Durie, 2006, p. 183).

Part 3
Methodology and preliminary studies

5 Methodology

The core research methodology aims to build a "thick description" of mathematical social life in the form of a working system for peer produced peer learning.

5.1 The importance of building a working model

The following chapters examine "*man-made information sources for the directive ordering of human conduct*" (Geertz, 1973, p. 251) under conditions of emergence. This work retraces the layers of mediated sociality that were reviewed in Chapter 4: however the methods that will be applied are generally quite different from those of the authors examined there. Chapter 6 will exhibit a method for modeling the social antecedents to learning using statistical analysis. This is not a conceptual model of learning, but a way of showing, on average, what kinds of mediated acts catalyze learning outcomes. Chapter 7 uses focus group and interview-based methods to try to understand what potential users of PlanetMath would consider to be pragmatic for their purposes. Chapter 8 describes the design and development of the Planetary system, which is intended to be feature complete relative to Noösphere, with some key differences that that make it more suitable for peer learning applications. In Chapter 9, motivations to use PlanetMath/Planetary are investigated through discourse with users, who are involved in thinking critically about the affordances and limitations of the software. Research project does not focus on the processes of proof invention, heuristic discovery, and good old fashioned textbook problem solving, with which mathematical social life is often occupied, but rather the matrix in which these activities can be embedded. This does not mean that day-to-day mathematical activities are construed to reduce to a determination from the macro level (DeLanda, 2006A, p. 5) any more than we would say that the theorems of Euclid or

Archimedes derive from the floor plan or scribal practices at the Library of Alexandria. A emergent multi-layered approach, after (DeLanda, 2011) engages mathematicians and others in thinking carefully and critically about the "*the basic mental infrastructure of mathematics*" (Thurston, 1994), which as Chapter 4 has shown, has important social and cultural dimensions. Frayling (1993) describes "*Customising a piece of technology to do something no-one had considered before, and communicating the results*" as an example of "*research through art and design.*" This is a crucial aspect of the approach taken here, however, as the current effort aims to produce an actor-oriented model, what is most important is that the development of a prototype opens the way to consultations with users about the customizations they would like to see made. The findings comprise key steps on the way to the "*thickest possible description*" (Ryle, 1971) – in this case a software system that the research subjects would not only recognize as a reasonable account of relevant social experiences in mathematics, but which at least some of them would ultimately find useful. The thematic focus of the project as a whole is on software development in a social context. This approach affords the opportunity to build and jointly critique a working model of mathematical social life.

5.2 Review of primary methods

Microgenetic analysis: studying the process of learning

Open online learning environments provide researchers with access to most if not all of the data that participants use to communicate with one another (Stahl, 2006); *cf.* [§4.2, p. 50]. These data can provide detailed evidence of learning and development. This kind of data – and the informal, *ad hoc* interactions that generate it – matches many of the ideas and techniques of *microgenetic analysis*. The microgenetic approach studies learning as a process, rather than the outcome of a process (Lavelli, Pantoja, Hsu, Messinger, & Fogel, 2005). The approach aims to examine moment-by-moment changes, often observed over a hort period of time, and often across a high number of separate observations, but not necessarily subject to the same staged treatment patterns found in longitudinal studies. Observations tend to be analyzed intensively, both qualitatively and quantitatively. In particular, microgenetic approaches have been used to take into account the social process of development in which individuals learn concurrently in a distributed fashion (Fischer & Granott, 1995). Broadly speaking, microgenetic analysis seems particularly well suited to our informal peer learning context, where a pre-test/post-test method for assessing learning quality would be inappropriate or unfeasible. The study in Chapter 6 can serve as a proof of concept illustrating the microgenetic measurement of learning, as adapted to the PlanetMath setting and dataset. Statistical analysis of this sort of data presents some unique challenges (Cheshire, Muldoon, Fran-

cis, Lewis, & Ball, 2007). One particular challenge for modeling is that some users may not log in for long periods at a time, and may be strongly motivated in short bursts of activity; *cf.* (Barabási, 2005).

Chapter 6 examines additions and modifications to the PlanetMath encyclopedia taking place between Oct 6, 2001 and Dec 16, 2010, along with forum posts and corrections made during this time. The population of interest is comprised of the 445 people who made edits to the encyclopedia in this time frame. For the purpose of the study, changes to the encyclopedia in which a contributer uses a new word (on a per-person basis) is the outcome of interest. The analysis is based on a recently developed statistical method that is well-suited to handle bursty dynamics (Teravainen, 2014).

Focus group and field work: Gathering user desiderata

Learning about and responding to users' design criteria is well studied in software engineering, where this is usually referred to *requirements analysis*.

> *Project participants or stakeholders may assign to the design object one or more goals – optative statements describing a change in the environment that the design object is desired to produce. A* ***requirement*** *is a feature of a design object that is necessary to achieve a goal.* [...] *More generally, suppose g may be achieved by n design objects* $(D_1, D_2, \ldots D_n)$ *each with features* $(F_1, F_2, \ldots F_n)$*. The set of requirements* R_g *may then be defined as the intersection of the properties* $(R_g = F_1 \cap F_2 \cap \ldots \cap F_n)$*.* (Ralph, 2013)

In Chapter 7, I focus on what Ralph (2013) terms *desiderata*, rather than requirements. Chapter 8 revises this material into a minimal intersection of features for which the term *requirements* is more appropriate. In initial fieldwork, I aimed to understand users' goals, especially, but not only, the ones that could be achieved using software. At implementation time, a minimal kernel of features was developed, although this had as much to do with the limitations of time as with the empirical structure of user goals.

Chapter 7 is built around a focus group with fellow postgraduate students, and a brief period of field work and interviews with domain experts. This chapter uses the paragogy framework from Chapter 3 to map out the dimensions of change that seemed most relevant to these individuals as representatives of their respective stakeholder groups (students, mathematicians, and researchers with an interest in mathematical practice). The choice of populations to work with approximately follows (Corneli & Mikroyannidis, 2012). Inspired by Geertz, I adopt an interpretive perspective.* I relied on the idea of coherent change as an interpretive principle. The questions I returned to with research subjects were "*Would you find this sort of tool suitable for use in your work?*

*"[I]*nterpretation in my view is fundamentally about getting some idea of how people conceptualize, understand their world, what they are doing, how they are going about doing it, to get an idea of their world.*" Clifford Geertz, quoted in (Panourgía, 2002); reprinted in (Panourgía & Marcus, 2008).

(And if not, how would it have to change in order to be useful?)" The aim at this stage was to engage stakeholders in brief critical discussions about the potential usefulness of PlanetMath, from which I could extract a range of "*optative properties*" (Zave & Jackson, 1997). Statements about PlanetMath were also, explicitly or implicitly, statements about the stakeholders themselves.

For the initial focus group, 6 participants were recruited via email sent to the postgraduate student mailing list at the Open University (which reaches a population of around 250 local postgraduate students), and with an in-person announcement at the weekly postgraduate forum at the Centre for Research in Computing. The respondents, who as it turned out, were all from the computing department and who were all known to me previously, met in one focus group. I was able to make an audiovisual recording of the one-hour meeting. After a short presentation about PlanetMath, participants were asked to discuss three sets of questions in a variant of the "fishbowl" format.* Participants were divided into two sub-groups of 3 persons, who then were seated facing each other, at either side of a table. Group A was comprised of individuals who self-identified as "more mathematical" and Group B of those "less mathematical" in terms of their research interests and background. First Group A discussed a set of questions, while Group B listened, and then Group B discussed these questions while Group A listened. This design was employed to help ensure that everyone had a chance to talk, and to let differing views develop in the relative secure context of in-group dialog. The three sets of questions were increasingly vague, aiming to elicit exploratory interpretation on the part of participants (Figure 7). Each of the two-part rounds lasted 15 minutes. Participants were also asked to complete a brief questionnaire, which included space for free-form feedback.

My field work with domain experts again began with a presentation about PlanetMath, this time taking part in a seminar series at the School of Informatics at the University of Edinburgh. My primary contacts, a mathematician and a researcher who studies mathematical practice, who had both agreed to do interviews with me, were in the audience. During my four day research visit, discussions often happened in public (for example, after the seminar, or at a restaurant), and in the first private interview, the interviewee preferred not to be recorded. During the four-day visit, I therefor relied for on a daily journaling protocol as a way to create a record of what I was learning. This protocol (Figure 8) was adapted from a set of questions in a follow-up email from one of the ("less mathematical") participants in the initial focus group. Selections from the video transcript from the focus group and sources for quotes appear in Appendix C, and brief sample excerpts from my field notes in Appendix E.

*HTTP://WWW.CO-INTELLIGENCE.ORG/Y2K_FISHBOWL.HTML

First set of questions ("Easy")

- Would you find this sort of tool suitable for use in your work or studies?
- If you're an instructor, how might you have to change your teaching or assessment strategies if PlanetMath becomes popular?
- If you're a student, how might PlanetMath change your study strategy?

Second set of questions ("Medium")

- How do you currently solve problems?
- What do you do if you get stuck?
- What tools and strategies do you use to learn?

Third set of questions ("Hard")

- Describe your learning style.
- How do you deal with people whose viewpoints are very different from yours?
- How important is it to you to "own" what you create?

Figure 7: *Questions for discussion, posed to participants in the initial focus group of post-graduate students*

1. What were your key hypotheses at the start of the day?
2. How did you analyze the data you collected?
3. How did the results change what you knew before?
4. Did any of your hypotheses get corroborated as a result of your data analysis? If so, which, why and how strongly?
5. Did any of your hypotheses get challenged as a result of your data analysis? If so, which, why and how strongly?
6. Did you create or encounter any new hypotheses during or after the day's events? If so, please specify what they are and how you are going to test them.
7. How did any of the hypotheses relate to learning?
8. In hindsight, which questions proved to be the most or least salient?
9. Followup studies?

Figure 8: *Outline of journaling protocol*

A basic grammar for interaction

Table 10 presents a summary of PlanetMath's *entity relation diagram* ("*ER diagram*"), showing the kinds of interactions that will be supported in PlanetMath/Planetary.* The research agenda is rooted in the idea of iterative development, whereby the basic interaction design will gain additional detail and expressive thickness.

Hence this central methodological proposition:

> A sufficiently extended version of the "grammar" presented in Table 10 should be able to transform mathematical texts and interactions into computationally accessible objects in a coherent way.

The five-fold division in Table 10 is again inspired by the paragogy principles from Chapter 3. Although the table provides a useful initial plan for implementation, from the point of view of modeling mathematical behavior, it is certainly not complete. For instance, one rather simple extension to the table would add subscripts to the arrows to code for the different agents involved – this would be needed for modeling the use case described in Figure 3 [§2.3, p. 31], for example.

To narrate the table's current entries: the encyclopedia, made up of interlinked articles and concepts provides the central/decentered shared context. The simplest kind of contextual glue is relatedness.† A link $A \overset{\ell_1}{\leftarrow} A'$ between two articles is only useful if the two articles are, indeed, *related*. Here, $A \overset{\ell_1}{\leftarrow} A'$ denotes a connection from A' to A via a standard hyperlink, whereas $A \leftarrow A'$ means that article A' has been added as an attachment to article A.

Context	Feedback	Quality	Structure	Heuristic
$A \leftarrow A$ $A \overset{\ell}{\leftarrow} A$	$X \leftarrow T$ $S \leftarrow R$	$X \leftarrow Q$ $A \leftarrow C$	$A \leftarrow P \leftarrow S$ $L \leftarrow A, P$ $M \leftarrow A$ $Q \leftarrow A$	$S \leftrightarrow H$ $G \leftrightarrow U$ $Q, T \rightharpoonup C, W, P$
A article ℓ link	X object T post S solution R review	Q question C correction	P problem L collection M classification	H heuristic G group U user W request

Table 10: *A paragogical decomposition of PlanetMath's activities: "production rules" in the grammar of mathematical behavior*

* *Cf.* Figure 1 [§1.4, p. 12].
† *Cf.* Nowak's rules of the evolution of collaboration [§4.4, p. 59].

Learning can happen in many ways on PlanetMath, but learning about learning seems most likely to be mediated by informal feedback, and developed in informal discussions. On the other hand, quality control develops through more structured peer review. Problem solving applications represent another layer of semantic structure. Deeper organization is driven by the use of heuristics, embedded in problem-solutions ($S \leftarrow H$), currently in a way that makes them hard to extract. Social heuristics involve purpose-created groups that solve problems together. Finally, these processes are accompanied by a curatorial or administrative process that mutates informal discussion into objects with more explicit meanings in the system ($Q, T \rightharpoonup C, W, P$). Time flows from left to right for all of these expressions. The table suggests various vectors of data on user activities that could be pulled out of the database: *new links between articles added*, *new questions asked*, *new corrections submitted*, *new problems proposed*, *new group memberships activated*, and so on. Rebuilt on this design, PlanetMath will become a large-scale recording device for interactions related to learning mathematics. In addition to mathematical relationships between articles and other objects, the "production rules" in Table 10 serve to code for various features of human collaboration. For example, by attaching a comment to an object, the user collaborates with the object creator by offering feedback. By attaching a correction to an article, the user collaborates not only with the original author, but more broadly with the readership, by helping to maintain quality and consistency throughout the encyclopedia. Attaching a problem to an article or organizing a set of articles and problems in a collection helps to structure the corpus, creating a small subarea with coherent meaning. People can use groups to collaborate as co-authors or in *ad hoc* teams. Collections and groups are also useful for specifying and sharing learning and production goals, and for breaking them down into smaller components. Chapter 8 describes the development of the Planetary system, which progressed along the lines indicated by Table 10, and further informed by the findings from Chapter 7. Chapter 8 uses Marshall McLuhan's (1989) tetrad of media effects to highlight the differences that the implemented features are expected to make.

Design patterns and emergent design

Peer production is not a panacea. On the one hand, "*many successful peer production projects exhibit strong inequalities of participation and deeply entrenched leadership*" (Shaw & Hill, 2014). On the other, many free/open software projects never attract collaborators at all: "[t]*he benefits of collaboration become something to understand, support, and work towards, rather than something to take for granted*" (Hill, 2011b). This challenge will be approached here in a methodical way through the use of *design patterns* to describe problems and develop solutions within a paragogical context. This application of design patterns extends David Cavallo's (2000b) thinking about the *emergent design* of learning environments, rendering it somewhat more systematic.

Title: Encapsulate the idea – possibly include a subtitle.

Definition: Explain the idea and the context in which it is meaningful.

Problem: Explain why there's some issue to address here.

Solution: Talk about an idea about how to address the issue.

Challenges: Talk about what can go wrong.

What's Next: Talk about specific next steps.

Examples, Illustrations, Objectives, and References are optional.

Table 11: *Pattern template from the* Peeragogy Handbook

Christopher Alexander first introduced design patterns as part of an holistic approach to architecture, urban planning, and construction (Alexander, Ishikawa, & Silverstein, 1977; Alexander, 1979). The design pattern methodology spread from architecture to software (Vlissides, Helm, Johnson, & Gamma, 1995; Coplien & Schmidt, 1995; Gabriel, 1996), and later, to other fields, including public affairs (Schuler, 2008) and education (Bergin et al., 2012). A relevant recent survey is included in (Kohls, 2013).

Alexander's patterns are presented in tree-like structure called a *pattern language*, ordered in a top-down manner from large-scale to small-scale levels of application, with each pattern presented in terms of a *picture*, a *context* (including links to relevant larger patterns), the *problem* that the pattern addresses, the *solution*, a *diagram*, and *links to smaller patterns* (Alexander et al., 1977, pp. x-xi). A relatively convincing implementation of Alexander's idea of patterns as a "*living language*" (Alexander et al., 1977, p. xvii) was realized with one of the earliest applications of wiki software developed by Ward Cunningham: the Portland Pattern Repository.*

Taking the idea of a pattern language as a point of departure, the *Peeragogy Handbook* (Rheingold et al., 2014) includes a catalog of patterns for peer production and peer learning that the authors discovered while working on the book. The patterns are presented using a template (Table 11) that is closely aligned to the structure coming from Alexander. Each pattern is essentially a short textual abstract, and describes an active dimension of the Peeragogy project. This emphasis on the active aspect is made particularly clear through the "What's Next" facet, which concretely links the patterns in the *Handbook* to the current activities being conducted within the aegis of the Peeragogy project. The *Newcomer* pattern is presented as an example in Table 12.

*HTTP://C2.COM/PPR/

Newcomer: Welcoming "beginner's mind."

Definition: Unless there is a new person to talk to, a lot of the "education stuff" we do could grow pretty stale. Many of the patterns and use cases for peeragogy assume that there will be an audience or a new generation of learners.

Problem: Some of the problems are well summed up with a quote:

> **Régis Barondeau**: I joined this handbook project late, making me a "newcomer." When I started to catch up, I rapidly faced doubts: Where do I start? How can I help? How will I make it, having to read more than 700 posts to catch up? What tools are we using ? How do I use them? Etc. Although this project is amazingly interesting, catching the train while it already reached high speed can be an extreme sport. By taking care of newcomers, we might avoid loosing valuable contributors because they don't know how and where to start, and keep our own project on track.

Solution: It is good to try to become aware of what a newcomer needs, and what their motivations are. Another quote can illustrate:

> **Charlotte Pierce**: Joe was working a lot on the book, and I thought "this is interesting hard work, and he shouldn't have to do this alone." As a Peeragogy newcomer, I was kindly welcomed and mentored by Joe, Howard, Fabrizio, and others. I asked naive questions and was met with patient answers, guiding questions, and resource links. Concurrently, I bootstrapped myself into a position to contribute to the workflow by editing the live manuscript for consistency, style, and continuity.

Challenges: Newcomers in the Peeragogy project have often complained about feeling confused, suggesting that our project roadmap may not be sufficiently clear, and that more work has to be done the project accessible. Even in the absence of actual newcomers, we need to try and look at things with a "beginner's mind."

What's Next: We recently revised the "How to Get Involved" page, listing the top ten sites we use. Another reasonable thing to post would be a top-ten list of activities, so that people can get an easier view on the kinds of things we do in the project.

Table 12: *The* Newcomer *pattern from the* Peeragogy Handbook

The main features of the Peeragogy pattern template are analogous to the paragogy principles [§3.1, p. 36]. The *definition* is meant to connect the pattern to a particular context; specifying the *problem* provides the opportunity to explain why there's something here to learn; a *solution* usually involves interactions with peers; *challenges* arise in practice due to the nonlinear nature of real-world problems; and writing down *what's next* indicates that after identifying a pattern, there is usually still work to be done.

This approach seems to be a novel interpretation of the design pattern methodology. For comparison: the Public Sphere project uses a detailed template to elicit input for their pattern catalog. They aim to distill existing, ambient, "*civic intelligence*" in an effective form.[*,†] However, although this pattern language continues to develop as a work in progress, the patterns are not explicitly linked to actions taken in the project. Table 13 presents a succinct overview of the Peeragogy pattern catalog, with references to the Minskian heuristics from Table 4 [§4.2, p. 53]. This helps to show how the Peeragogy patterns are relevant as a set of heuristics for social problem solving. Further details are in (Rheingold et al., 2014).[‡] The Peeragogy project continues the tradition of reflexive research-through-writing that was used by Tomlinson et al. (2012). Both of these efforts used their own collaborative writing and research activities as primary sources of data. Building on this bootstrapped material, the application here is comparatively straightforward. In particular, Rheingold et al. (2014) also deals with antipatterns – "[f]*requent occurrences that are not desirable*" – which are not considered here.

In Chapter 9, feedback arising in the user evaluation of PlanetMath/Planetary is analyzed in terms of the Peeragogy pattern catalog, and the catalog is extended with three additional patterns. In the work reported in Chapter 9, PlanetMath users were not involved directly in doing pattern-based analysis or coming up with new patterns: rather, I wrote new patterns based on what I learned in our interviews. In preparing the analysis, I used a "path metaphor" similar to that employed by Christian Kohls (2010, 2011), who describes patterns in the language of constrained optimization problems, considering the initial state, end state, and forces acting.

At the level of process: I successfully recruited 6 people who completed a 10 day study on PlanetMath that took place two months after the launch of the new software system.[§] The participants were 3 PlanetMath users and 3 mathematics postgraduate students from the Open University. Two of the PlanetMath users had prior familiarity with PlanetMath/Noösphere, and two Open University Masters students were familiar with online distance learning. Four participants use mathematics professionally.

[*] HTTP://PUBLICSPHEREPROJECT.ORG/PATTERNS/CHEAT-SHEET.PHP

[†] HTTP://WWW.PUBLICSPHEREPROJECT.ORG/TEMP/YOUCANHELP.PHP

[‡] And in the online version of the book, at HTTP://PEERAGOGY.ORG/PRACTICE/.

[§] A seventh participant was recruited from the *Peeragogy in Action* Google+ community, but he dropped out of the study before completing the planned tasks or an interview.

We *simplify* things for a **Newcomer**.

We *change focus* by using a **Roadmap** to guide us from one step to another. Every roadmap is associated with **A Specific Project**, and the specificity is important. In addition, the project's **Heartbeat** leads us to let go of our focus at one moment, and resume with another point of view later.

We *change description* first of all by having a **Wrapper** who describes the new state of the project. For the Peeragogy project, that often meant summing up the high points that we saw over a given period of time. It seems possible that with a rich enough **Pattern Language**, the description would itself be made in terms of patterns.

We *divide* work up not only "horizontally" among different **Roles**, but also "temporally" by using the **Roadmap**. Someone who is moving ahead with the Roadmap is likely to be "working at the cutting edge".

When we *find an analogy*, we are basically **Creating a Guide** of some sort. This can be used as a form of "exploration", as we look at how one form of engagement may or may not map onto other forms of engagement.

When we *ask for help*, we may avail ourselves of some **Moderation** service that will decide how to deal with our request. One simple way to ask for help is **Polling for Ideas**. Obviously once we start to get help, we're working in a regime of "collaborative effort".

If you *know the answer*, then you may be able to reuse it (which is the basic idea of **Use or Make?**). Someone who knows the answer and who is good at self-explanation may also have a good idea about how to get from the current state to the goal state; alternatively, this may be broken down into steps in a sub-**Roadmap**, and moving from step to step would then illustrate "progressive problem solving".

It is important to *give it a rest* so as not to over-exhaust oneself, busting one's own **Carrying Capacity**, or, alternatively, overwhelming the group.

It seems that one of the things that *experts do* is **Discerning a Pattern**. This allows them to simplify their processing.

Finally, again, if we *know why it is hard*, then we may be able to **Create a Guide** that will help get around, or at least better cope with, the difficulty.

Table 13: *Overview of the central patterns in the* Peeragogy Handbook

(Step 1) You'll join an hour-long online focus group, where I'll brief you on the tasks for the next step, and give a more detailed description of the study, as well as gather some initial feedback about the site in an open discussion.

(Step 2) You'll try out the new system features that we've developed – this can happen as you have time, over a week or two. We'll be able to talk about issues or concerns as they arise, in the PlanetMath forums.

(Step 3) You'll meet me with me for a small group or one-to-one interview to talk about your experiences with the system; this will again take less than an hour.

Figure 9: *Steps in the user evaluation (from an email sent to 30 high-scoring and recently-active PlanetMath users)*

The steps in the study are indicated in Figure 9. In an initial meeting held via Skype, I walked study participants through a set of screenshots from PlanetMath/Planetary (presented here in Chapter 8). As a set of tasks to complete over the next 10 days, I asked participants to try each of the features described in the screenshots. I asked users to *introduce themselves, to describe their mathematical background, and to say what interested them about the study.* I suggested that they work together in case of any difficulties, attempting in this way to foster a peer learning culture within the group. Participants who didn't yet have PlanetMath accounts made them at the start of the study. Over the next 10 days, most participants tried many of the tasks, sometimes posting comments about their experience in the PlanetMath forums, and sometimes emailing me directly to ask for help. After the 10 day period had elapsed, I conducted individual follow-up interviews.

As in the initial study with stakeholders, the main question I focused on in the interviews was: *would PlanetMath be relevant to the way you study and do mathematics?* I also tried to elicit ideas that might help to improve the site, making use of shared familiarity with the system.

During the interviews, I took graphical notes and made audio recordings (when possible). Interview data was analyzed twice: first on a thematic basis, by connecting user feedback to the main dimensions of the design of PlanetMath/Planetary discussed in Chapter 8, and again by looking for matching patterns in the Peeragogy pattern catalog (Table 13). The idea in the pattern analysis was that either the existing catalog of Peeragogy patterns would describe the topics under discussion, or new patterns would emerge with links to the existing patterns showing how they are similar or different. In either case, a contribution will have been made to enhancing the usefulness and descriptive power of the pattern catalog.

5.3 Summary

The patterns that guide behavior in the PlanetMath context are related to historical mathematics practices, but are understood to have features that are emergent in the new setting. This invites a "DeLandan" perspective on culture that looks at the emergent dynamics within and across several levels of analysis. The primary methods are statistical analysis of historical data, qualitative field work, implementation of a software system for collaboration, and emergent design.

Several novel methodological points were introduced, including the simple idea of using vocabulary growth as part of a statistical model of learning in a technical domain, and the more complex idea of using design patterns to represent the heuristic, and emergent, aspects of paragogical projects. Paragogy was useful in developing an initial entity relation diagram for PlanetMath/Planetary, and new schemes for analyzing qualitative data. In this way, paragogy has been baked into the design of both the software system and the research process.

The overall aim is to do research through iterative development within an emergent design framework. A key limitation is that the following chapters are only *one* research cycle. This work necessarily focuses on the dimensions which are currently accessible to study, for example, "vocabulary use" as opposed to "heuristic use" – as well as the most basic technologies and use cases. The primary merit of the approach is that it opens the way to a participatory, computer-mediated, study of mathematical culture.

6 An Analysis of Learning Interactions in PlanetMath/Noösphere

The first study focuses on understanding the effects of peer support on learning in the course of encyclopedia authoring activities and forum discussions on the original PlanetMath.

6.1 The fundamental problem of instrumentation

This chapter looks at what causes a change in propensity to use new vocabulary words among contributors to the PlanetMath website. In addition to new vocabulary, in practice, there are of course many other nonlinguistic, embodied, behaviors that people learn when they learn mathematics. The statistical methodology employed here would be equally applicable to the effect of behavior on word use, or the effect of word use on behavior. The focus is not on the internal or embodied organization of learning, but on the social covariates, antecedents, and possible precipitating causes. The particular strength of the method that was employed here is that, unlike standard time series methods, this approach is designed to filter out effects that are due to "calendar time", like seasonality, leaving only effects that are tied to the "gap time" between events. The technique works with any pair of time series vectors, one representing the *treatments*, and one representing the *outcomes*.

From a qualitative perspective, describing a given antecedent, for example, giving instruction, as a "*cause*" of learning is rather thin: it may be better to speak of statistical effects as "*catalysts*"; *cf.* (DeLanda, 2006B). After all, we could always go back further, looking for earlier and deeper contributing causes. What the method used here shows is that there was a significant social contribution to learning in PlanetMath/Noösphere; and it shows the differential effect of two different catalysts.

Due to it's generality, the method exhibited in action here is a candidate solution to the fundamental problem of instrumentation: *how do we know if it's working?* For example, if we were to use PlanetMath to make recommendations, as in the use case from Figure 3 [§2.3, p. 31], we would want to know that the recommendations were having the desired effect. The method demonstrated here could help.

6.2 Design and related work

The study aims to demonstrate that peer interactions in PlanetMath/Noösphere had a tangible and measurable effect on learning outcomes. New types of interactions may produce vastly different, and stronger, effects. It will be important to have a framework for analysis that can be used to gauge the effect of these interactions on desired outcomes.

The study offers a preliminary quantitative analysis of learning on PlanetMath, using legacy data from the site's first 10 years of existence. Even though PlanetMath was not designed to function as a learning environment during this phase, by design it had an explicit "*pedagogic slant*" [§2.1, p. 21] andr it is reasonable to hypothesize that people were, in fact, learning through their interactions on the site.

It is important to mention that during this time period, the growth of the PlanetMath encyclopedia, and, in its metadata layer, an associated mathematics lexicon, was the key measurement of success. Practices of peer review and norms like "one definition per concept" and "proofs should be self-contained" afforded a degree of integration and rigor that would not be directly possible with a fully-distributed information source (like the blogosphere).

Learning new things and sharing existing knowledge were two possible motivations for contributing to the encyclopedia. In some sense the latter motivation would have to be typical for encyclopedia authors – although the two motivations are not mutually exclusive, as expressed in the adage "*The best way to learn is to teach*" (Luk & Wang, 2012).

Distinguishing between the use of new words at the personal and community levels might help to distinguish between these two motivations. Someone who often uses new words on a per-person basis that are not new on a community-wide basis may be a more typical learner. Someone who consistently adds globally new words is more likely to be an expository writer or researcher.

This study focuses on the use of new words on a per-person basis, without distinguishing the words that are globally new from those that are only personally new. The analysis focuses on the immediate catalysts for the use of a new word, comparing a self-

started effect (posting in the forum) and an other-started effect (receiving a correction). For the purpose of the study, any use of a personally new word is thought of as a positive outcome, and considered to indicate learning in some form.* Even if in many cases this is learning in the course of practicing something that one already knows reasonably well, it is interesting to understand the social catalysts for such practice.

In order to facilitate the comparison between the two effects, I processed all of the data describing contributions to the PlanetMath encyclopedia from the first 10 years of the website, and found the timestamped changesets where each user first uses a term that is part of PlanetMath's technical lexicon. These technical terms include the titles of articles, as well as synonyms noted by the author in article metadata. Along with this word usage data, timestamped entries in the database showing when authors had either posted in the forum or received a correction.

This data was then analyzed using the recently developed computational method from Timothy Teravainen's PhD thesis *Semiparametric Estimation of a Gaptime-Associated Hazard Function* (Teravainen, 2014), discussed in overview in (Wu et al., 2011). This work falls within the field of statistics known as survival analysis; *cf.* (Fleming & Harrington, 2011). The method estimates the increase in likelihood associated with the specified outcome (here, using a new word) after the treatment event (here, either the self-initiated treatment of posting in the forum, or the other-initiated treatment of receiving a correction). The change in likelihood is considered in comparison to a otherwise equivalent user who did not receive the treatment, and the results are averaged across all contributing users and all time.

For comparison: Albert-László Barabási's paper on "The origin of bursts and heavy tails in human dynamics" (Barabási, 2005) uses a large collection of email correspondence as a data source. Barabási's models use power law functions, which are generalized by the Ornstein-Uhlenbeck process at the core of the computational model employed here (Behme, 2011, p. 12).

There are many relevant techniques for dealing with social networks and time series data; *cf.* (Rajaraman & Ullman, 2011, Chapter 10), (Quenouille, 1949). Standard cross correlation is the most typical approach to studying time series data, but it cannot distinguish between gap time effects and calendar time effects. For this study, the typical gap time is of interest, rather than seasonal effects. Many modeling efforts consider only the relative *sequence* of events, discarding the timestamps – *e.g.* Abbott (1995) – but this approach is not well-suited to studying activities that are distributed over long periods with bursts of activity.

* *Cf.* Leung [§4.2, p. 51], Halliday [§4.2, p. 52] and Minsky [§4.2, p. 53] for some of the considerations involved with using vocabulary growth as a proxy measure for learning.

Various corpus techniques could be used to extract meaningful patterns from a text (Pustejovsky, Anick, & Bergler, 1993). Any refinement to the input data would be compatible with the statistical model used here, as long as the data can be mapped to a time series. Work with named entities is particularly straightforward and well-studied (Nadeau & Sekine, 2007). The LIWC, mentioned in Chapter 4 is an example of this sort of approach, and focuses on psychologically meaningful words, including pronouns, and words related to emotions and causality; *cf.* (Pennebaker et al., 2001).

Methods

The last ten years of versioned encyclopedia articles were loaded into Git, which made it relatively easy to extract line-by-line differences between versions, using `git log` and `git show`. These change sets were analyzed using a term spotting approach, with technical terms drawn from the thesaurus-like metadata from PlanetMath's encyclopedia articles. This was done using *ad hoc* code implemented around a regular expression based search.* The functionality is similar to that of PlanetMath's autolinking tool, NNexus (Gardner, Krowne, & Xiong, 2006), and with a few changes to NNexus, the method used here could be readily adapted to make use of transcripted NNexus data. Subject to the general caveats above, that detected words may in many cases be exposition events rather than elementary learning events, the outcome of interest is any encyclopedia edit that includes a new word from the lexicon (per-user, considered relative to the history of their previous contributions to the encyclopedia). During the time period under observation (Oct 6, 2001–Dec 16, 2010) for the population of 445 "committers" under consideration – exactly the population of persons for whom we have edits on record – there were a total of 8051 forum posts made, 14064 corrections received, and 3867 modifications to the encyclopedia in which any non-zero number of new (per user) technical terms were detected.† The method applied to analyze this data was a slight variation on that studied in Teravainen (2014), in which the primary application was to sequences of events with a statistical self-retriggering property that did not present the need to distinguish between "treatment" and "outcome". Here, the computation first identifies gaps between treatment and outcome events, and then aims to estimate the effect that the most recent previous treatment had on a given outcome. It does this by fitting the data to a physical model with two primary dimensions: an initial impulse and a damping effect (an "Ornstein-Uhlenbeck process"). Teravainen considers stochastic generalizations of this model that were not applied in the computations here.

*HTTP://METAMESO.ORG/~JOE/MATH/THESIS-SUPPLEMENT/

†PlanetMath defines quite a few more than 3867 terms, and one might expect an aggregated per person usage statistic to be still bigger than the total number of terms available. The reason for a lower figure may be accounted for by clarifying just what was counted as a "usage event." First, articles can define multiple terms, and if multiple terms are introduced in one editing pass, then this would only count as one instance of "new word usage." Second, many articles have titles like "*proof of . . . theorem*" which never appear in the article's body, and which consequently wouldn't be detected in the changesets.

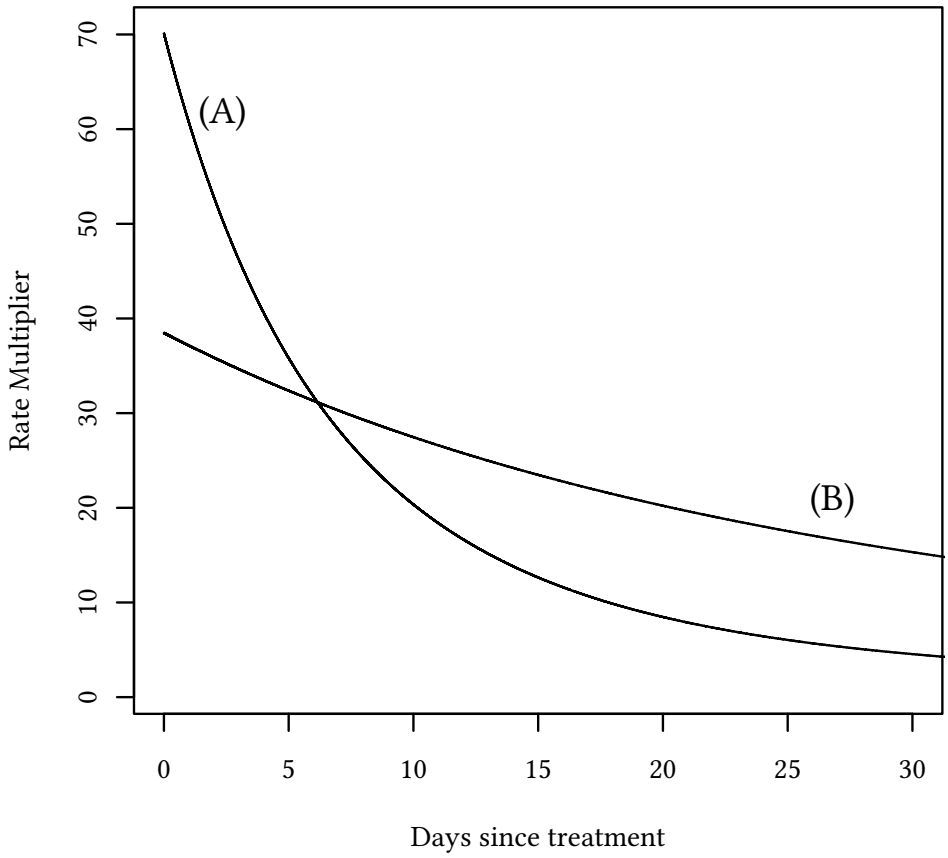


Figure 10: *A plot showing the differential effect of (A) an exogenous treatment* (receiving a correction)*, and (B) an endogenous treatment* (posting in the forum) *on* using new words in the encyclopedia

6.3 Estimating effects

Intrinsic self-motivation is a well-known catalyst for learning. It is interesting to consider the differential effect on learning outcomes from stimuli that are self-driven from stimuli that are initiated by peers. Two trajectories along these dimensions are examined:

Trajectory A. The effect of *corrections received* on *new technical terms used.*

Trajectory B. The effect of *posting in the forum* on *new technical terms used.*

Trajectory A is other-motivated; Trajectory B is self-motivated, but occurs within a social context. As shown in Figure 10, Trajectory A has a strong initial effect, but wears off relatively quickly, whereas Trajectory B has a much more moderate effect,

and wears off slowly. This suggests that people who are motivated to post in the forum are also motivated to keep adding new concepts in encyclopedia articles – and it also shows the strong value of peer support even among this self-selected group of highly self-motivated people.

6.4 Potential refinements

In the first place, without considering any semantic extensions to the underlying model, it would be useful to have confidence intervals around the curves in Figure 10. Such estimates are forthcoming.*

It would be useful to extend the model to study several different treatments in parallel; this could help to tease out the differential effect of a socially-motivated post in the forum *versus* an edit to the encyclopedia. At the moment, the model compares a given treatment to "no treatment"; it would be useful to be able to run more elaborate simulated control studies.

The current model does not make use of the "networked" aspect of the PlanetMath encyclopedia or the underlying social network. A more ideal model of an individual's learning patterns would consider the geometric features of the system that they are studying. It seems reasonable to hypothesize that person would be more likely to learn terms that are near terms that they already know, and especially likely to (serendipitously or opportunistically) learn things that are adjacent to topics they are currently working on. A simple prior distribution could be employed, saying that, all else equal, terms are most likely to be learned in proportion to their PageRank, or inversely in proportion to their mathematical depth (which could be estimated by other graph-theoretic distance from known elementary concepts).

Apart from modeling issues to do with personal learning history, the modeling problem is complicated by the fact that people are different from one another in ways that have nothing to do with the domain. One possible adaptation of the current scheme would use a Dirichlet prior that takes into account the fact that there are "different kinds of people." For instance, we could categorize users according to the apparent strength of their intrinsic motivation. Some users may require more attention if they are to stay involved. More broadly, we might want to consider individuals with a range of features, per (Broderick, Mackey, Paisley, & Jordan, 2011).

An alternative interpretation of the PlanetMath data would come from transposing the matrix, so that instead of looking at new-terms-per-user, we could look at new-users-per-term. This approach would useful for studying community-level linguistic change.

*Timothy Teravainen, personal communication.

Vocabulary growth has previously been studied with the aim of predicting user migration. According to Danescu-Niculescu-Mizil, West, Jurafsky, Leskovec, and Potts (2013), "[a]*fter an initial period of adaptation to the language of their community* [...] *individuals' language patterns slowly rigidify until the moment they abandon the community.*" It would be interesting to see whether there are explicit strategies for lifelong learning in a deep domain like mathematics that help maintain user interest.

It would also be interesting to look at the developmental course of learning: do individual users undergo intensive periods of learning, similar to the burstiness studied by (Barabási, 2005)? Do users learn new nonlinguistic behaviors along with new vocabulary, per (Iverson & Thelen, 1999)?

6.5 Summary

This chapter has employing a new statistical method to estimate the effect of two different treatments on learning outcomes. An analogous study could be performed in any context where learning can be modeled by vocabulary growth. This includes technical domains, and potentially any book with an index.

The significance of the approach is considerably more general, since, as suggested in principle by (Wu et al., 2011) and now illustrated here, the same techniques can be used to model the effect of any time-delineated series of "*blip treatments*" on a time-delineated vector of outcomes.

The techniques can be applied directly to study discrete online interactions that are likely to have effects on learning outcomes or other valued goals. With further refinements, this sort of analysis could inform varied diagnostic and recommendation services. Although these issues are not developed further in the thesis, this chapter does illustrate the potential usefulness of gathering varied data on user interactions, and this point is taken up again in Chapter 8.

7 Surveying Stakeholder Perspectives

The second study employs qualitative methods to build a richer picture of the stakeholders in a peer learning environment for mathematics.

7.1 Overview

The purpose of this study was to understand the perspectives on PlanetMath's rebuild from several likely user and stakeholder groups – students, mathematicians, and "reflective practitioners." The latter group is understood to include philosophers, social scientists, educators, and computer scientists whose work contributes to an "*inquiry into the epistemology of practice*" in the mathematics domain; *cf.* (Schön, 1999, p. viii).

Asking potential PlanetMath users what they would expect from a "peer learning environment for mathematics" offers the opportunity to learn something about what they think about mathematics *per se*, and to understand the role that computer-mediated communication plays in shaping their perspectives.

I have endeavored to convey the understandings that my research subjects offered in terms of the meanings that are operant for them in our conversations. The idea of PlanetMath constitutes a minor perturbation to the life-worlds of these research subjects; their framing of "the question" of PlanetMath is of particular interest.

> *A meaning-reaction is* [...] *a fitting adjustmental response which individuals acquire through the direct influence of the surrounding objects and conditions. It is in this way that a meaning-reaction becomes the means for bringing about an especially fitting adjustment of the person to his surroundings.* (Kantor, 1921)

In some cases, the subject's responses to PlanetMath, and what it represents, were enthusiastic – and in other cases, highly skeptical. Wherever these responses were not entirely explicit (whether they were favorable or unfavorable), I have expanded the discussion to help put them in context. For example, perhaps the most "interpretative" move on my part was to point out the connections between students' explicit mention of *trust* and the unspoken issue of *credit*.

At this point, it is not entirely appropriate to think of the collateral adjustments that are indicated for PlanetMath as "*requirements*" in the typical engineering sense; I have called them "*desiderata*," following (Ralph, 2013). The feedback elicited at this stage points to long-term goals that are not wholly realized by the implementation work described in Chapter 8. At the same time, the work presented in this chapter usefully conveys a sense both that this implementation work is "on track," and a meaningful indication of what it is pointed towards.

This research is based on a focus group I ran with 6 postgraduate students in computing at the Open University, and a 4 day research visit to Edinburgh, which was spent partly at the Department of Informatics at the University of Edinburgh and partly at Heriot-Watt. In Edinburgh, my primary informants – who have agreed to be non-anonymous in this report – were Murdoch James Gabbay and Alison Pease. I scheduled my visit at a time when I was also able to speak with Ursula Martin, Timothy Gowers, Mohan Ganesalingam, and Andrew Aberdein, and this chapter benefits from those discussions. From a practical standpoint, a standard of non-anonymity among the professionals I spoke with is justified because connections to published work are discussed in the chapter; from an ethics standpoint, this choice was seen as justifiable, since the points discussed were not contentious or especially personal. Had I been interested in a detailed ethnographic description of the lives of these informants, I would have opted to disguise the responses with pseudonyms.

Although the fieldwork methods used in the two settings were somewhat different – a "fishbowl" style discussion around my prompts in the student focus group at The Open University, and several days of relatively immersive discussions with professionals in Edinburgh [§5.2, pp. 71-73] – I have presented the findings in a uniform format. Inset quotes are paraphrases of things that respondents said; the relevant issues are then expanded in my own words. References [C1]–[C15] point to supporting data in Appendix C. The conclusions presented in the text were often, but not always, arrived at jointly with the research subjects.

The findings are presented in detail in the sections that follow, where they are mapped to the paragogical framework of dimensions of change from Chapter 3. A brief "composite" overview is presented just below.

Student

As long as their university courses provide everything that current students are seeking, they are unlikely to use PlanetMath, even as an ancillary tool. And yet it seems clear that PlanetMath could be used as a form of alternative credentialization – through a transparent record of contributions, including problems solved – or as a textbook, or even as a course management tool.

Mathematician

For a mathematics researcher, contributing to public discussions about mathematics is not typically done directly for the purpose of "getting credit" (as in, something to put on one's CV), but rather because it helps with research. There are many ways this could be better supported, both on PlanetMath and with tools that work across the web. Working in the capacity of instructor, a professional mathematician needs to help students learn relevant social as well as technical skills. One way to encourage this is to make suitably structured peer interaction a required component of courses. Some of these interactions could take place online.

Philosopher

Philosophers of mathematics working within the empiricist tradition are interested in the ways in which understanding arises. This can be studied using various kinds of models, ranging from heuristic frameworks imposed over data generated through human interaction to computer simulations. Insights gained through this work can help to improve education, tool design, and can also contribute to the field of artificial intelligence. PlanetMath could be both a source of useful information, and a consumer of the philosopher's insights.

7.2 A student perspective

Changing the nature of the space The issue of trust was raised even before I had shared any details about which factors might already exist that might instill trust. This minor incongruity suggests that the issue of trust might be conflated with another deeper issue. I suspected that this was the issue of credit. In a university course, students can trust their instructors and TAs to adhere to clear contractual obligations. They can trust that if they put forth a reasonable focused effort, they will earn an appropriate mark. So it is clear that in the traditional university setting, trust and credit are closely linked. However, without explicit attention to creating suitable mechanisms, in online peer production systems, neither trust nor credit is there

> *"Can we trust the people who are contributing?"* [C1]

by default. Accordingly, systems for distributed production, including Q&A sites, open source software projects, and online encyclopedias – some to better effect than others – have all created their own mechanisms for building trust and maintaining quality [§4.5, p. 61]. It seems that trust and credit go well together online, as well as offline. If it is easy to see what someone has contributed, and perhaps what others thought about it, then it's also reasonable to expect their engagement will continue roughly in the same vein.

Changing what I know about myself Small moves with continuous feedback at each stage could help build the user's confidence. The feedback may reveal that they know the answer to the problem, for example – and if not, taking small steps means that the user is never too far from what they do know. Indeed, mathematics naturally breaks down into small steps, but it may not be obvious how this works for a beginner. Learning *how* to break a task down into small, individually-masterable steps is a useful skill, and can help make mathematics less daunting (Mighton, 2003).* Students also need to learn persistence, but persistence through many small steps is one route to this.† More broadly, teaching that mathematical ability can be developed through hard work tends to have a lasting positive effect (Dweck, 2000, 2007).

"Does PlanetMath provide support for progressive problem solving?" [C2]

Changing my perspective In what might be called instrumental problem solving, some piece of information is needed as soon as possible; it is of fleeting interest and limited application. In non-instrumental problem solving, the needs are in some sense simulational rather than informational. What is at stake is building a lasting mental model of how things work. The nominal use of an encyclopedia is for lookup – the encyclopedia can play the role of the model. On the other hand, peer interactions open up the world of non-instrumental problem solving. It would be possible to make a further distinction and say that instrumental problems can be solved *transactionally*, whereas non-instrumental problems can only be solved *interactionally*.

"When I'm programming, I look up error messages online; but in my Calculus course, I don't want to ask a question until I've really thought a lot about the answer." [C3]

Changing content or connectivity Participants had many ideas about how the site should give feedback – ranging from having hints available to building rich "*ontology of problems*." It

"Are there good mechanisms for getting (and giving) feedback?" [C4]

* *Cf.* Gowers [§4.2, p. 50] and Leung [§4.2, p. 51].
† Jim Pitman, personal communication

occurred to me that since they had many creative ideas about this, in the long run it would be as important to be able take new feedback on board as to give it out. A rich give-and-take of feedback gets to the crux of (non-provisionist) peer learning. Tools like Get Satisfaction* are the typical way to elicit and record users' ideas about system features. Of course, actually acting on the feedback tends to be the real sticking point, and the only way to keep feedback coming. Transforming a flow of often highly transient attention into usable code calls for deep consideration of "*the match between cognitive processes and the statistical structure of the environments in which they function*" (Dhami, Hertwig, & Hoffrage, 2004) – across rather different populations. A project usually will not look the same to programmers and end users; *cf.* (Aboulafia, 2012, §5).

Changing objectives One might hypothesize a class of environmentally-distributed problems that are too complicated to simulate easily, and for which the environment is the best model.† On this basis, an economics of problem solving begins to unfold that is reminiscent of Coase's (1937) theory of the firm. There is a class of problems that is best solved "in-house," by interacting with and developing one's personal mental models – because building the mental model is the real point. There is a class of problems where the answer can be retrieved from one's existing mental model. This is the standard case of *knowing* – or what might in a litigious frame of mind be called "intellectual property." There is a third class of problems that are best solved through a transaction with the environment, by lookup or search – in this case, the only aspect of the mental model that matters is knowing where, or how, to look. This is the connectivist's actionable knowledge.‡ And there is a fourth class of problem which is again best solved interactionally, but this time, by interacting with others after the model of the commons, rather than that of the firm – in a meeting of minds. In practice, it seems there will always be a degree of sociality, in the sense of being "*betwixt and between*" [§1.3, p. 8] – a savant will think for a moment before answering, for example. The relevance for PlanetMath is that some people will want to follow relatively well determined channels which offer reasonably assured results, whereas others will want to experiment with new models, or tackle problems with greater risks. Outside of compulsory education, the student is typically someone who wants to earn a credential of some sort, which puts some constraints on the degree of newness. The "standard" methods should be low hanging fruit for PlanetMath.

> *"Some people are more social in their problem solving than others – and some problems lend themselves better to social problem solving."* [C5]

*HTTPS://GETSATISFACTION.COM/

†*Cf.* Clark (1998), also Feynman (1982).

‡*Cf.* Siemens [§4.1, p. 47] and Krowne [§4.5, p. 60]

7.3 A mathematician's perspective

"It would be useful to have better systems for giving and receiving feedback." [C6]

Changing the nature of the space / Develop empirical studies and a critical apparatus As Jamie explained to me, the relevance of changing communication technology, for a mathematician, can be discussed by analogy to considerations about a very different sort of technology: a commuter bicycle. A bicycle with a hybrid design and an internally-geared hub rides about 20% slower than a bicycle with true road geometry running a derailleur system. A communication technology can have similar beneficial or detrimental effects concerning knowledge transfer and synthesis. As this is the mathematician's bread and butter, it is a subject of some considerable concern. These concerns are not limited to computer mediated communication, however. A mathematician is sensitive to problems in exposition and direction – problems that are associated with an even more basic communication technology, language. In the seminar room, the audience asks questions both out of curiosity and to prompt the speaker. Over coffee, a non-expert may help to improve a research paper with a naive question. These are the kinds of interactions that designers of computer systems for mathematical communication need to consider. They are mostly missing from current systems, such the popular e-print repository ArXiv.org. * Jamie would like to being able to point out a problem in "*paragraph 7*" of a given text – or point to a completely different resource that does a better job.

"Highly-abstracted and non-expository articles are hard to understand: they suffer from a kind of impenetrable quasiformality." [C7]

Changing what I know about myself / Find companions for the journey We looked at the definitions of the mathematical term *ultrafilter* on PlanetMath and on Wikipedia (see Figure 11). In PlanetMath's definition, an ultrafilter is defined to be *filter* with certain properties. Wikipedia's definition describes the object first in terms of basic features of sets. It then develops several different equivalent conditions, and finally gives a definition in terms of filters. The PlanetMath definition takes up roughly the same amount of space as Wikipedia's "preliminaries", and manages to squeeze in a terse treatment of "types of ultrafilters." However, after the preliminaries, the Wikipedia article goes on to deal with *completeness*, *generalization to partial orders*, *types and existence of ultrafilters*, *applications*, *ordering on ultrafilters*, and *ultrafilters on* ω. It also provides some interesting-looking references, for instance, to a paper on "Arrow's Theorem and Turing Computability". In short, the Wikipedia definition appears to be superior to the PlanetMath definition in all the relevant ways: it is relatively intuitive, complete, and

*HTTP://ARXIV.ORG/

A COLLECTION $\mathcal{U}$ of SUBSETS of X is an *ultrafilter* if $\mathcal{U}$ is a FILTER, and whenever $A \subseteq X$ then either $A \in \mathcal{U}$ or $X \setminus A \in \mathcal{U}$. Equivalently, an ultrafilter on X is a MAXIMAL filter on X. More generally, an ULTRAFILTER OF A LATTICE is a maximal proper filter of the lattice. This is indeed a generalization, as an ultrafilter on X can then be defined as an ultrafilter of the power set $\mathbb{P}(X)$.

Types of ultrafilter For any $x \in X$ the set $\{A \subseteq X \mid x \in A\}$ is an ultrafilter on X. An ultrafilter formed in this way is called a *fixed ultrafilter*, or a *principal ultrafilter*, or a *trivial ultrafilter*. Any other ultrafilter on X is called a *free ultrafilter*, or a *non-principal ultrafilter*. An ultrafilter on a finite set is necessarily fixed. On any infinite set there are free ultrafilters (IN GREAT ABUNDANCE), but their existence depends on the Axiom of Choice, and so none can be explicitly constructed.

An ultrafilter $\mathcal{U}$ on X is called a *uniform ultrafilter* if every member of $\mathcal{U}$ has the same cardinality. (An ultrafilter on a singleton is uniform, but this is a degenerate case and is often excluded. All other uniform ultrafilters are free.)

HTTP://PLANETMATH.ORG/ULTRAFILTER

Given a set X, an ultrafilter on X is a set U consisting of subsets of X such that

1. The empty set is not an element of U.
2. If A and B are subsets of X, A is a subset of B, and A is an element of U, then B is also an element of U.
3. If A and B are elements of U, then so is the INTERSECTION of A and B.
4. If A is a subset of X, then either A or $X \setminus A$ is an element of U. (Note: axioms 1 and 3 imply that A and $X \setminus A$ cannot *both* be elements of U.)

A characterization is given by the following theorem. A FILTER U on a set X is an ultrafilter if any of the following conditions are true:

1. There is no filter F finer than U, i.e., $U \subseteq F$ implies $U = F$.
2. $A \cup B \in U$ implies $A \in U$ or $B \in U$.
3. $\forall A \subseteq X$: $A \in U$ or $X \setminus A \in U$.

HTTP://EN.WIKIPEDIA.ORG/WIKI/ULTRAFILTER

Figure 11: *Definitions of "ultrafilter", from PlanetMath and Wikipedia*

well-referenced. There could still be some specialized look-up purposes for which the PlanetMath is better – but one thing it does not provide is intuition. This is not to say that the PlanetMath article is "wrong." It may well be that after unwinding the recursive definitions, and expanding the attached articles, one could come to a clear and complete understanding of filters and ultrafilters.* A "*quasiformal*" style of writing is much more prevalent on PlanetMath, although it is not absent on Wikipedia. Wikipedia's "anyone can edit" policy tends to produce articles that are "good enough" on average (Krowne, 2005). It seems fairly likely that critical feedback about PlanetMath's ultrafilter article, of the kind described here, hasn't made it to the author(s) yet. However, the problems may more properly lie at the platform level. On PlanetMath, it is possible to create

*However, for technical reasons, attempting this wouldn't be so easy. The word *filter* in the PlanetMath definition was autolinked to the definition of the more specialized concept *lattice filter*, rather than the actual definition of a filter, putting the reader on a bit of a chase to find the relevant definitions.

an account, submit a correction, wait for the author to respond, and then check their response – but this workflow runs counter to the design principle that states "*with each click you require, you lose half your audience.*"* On Wikipedia, you could just make the change directly. Flaws in exposition point to the salutary (but missing) effect of contact and discussion with other people.

Changing my perspective / Work with real users An instructor who would like to make use of a peer pedagogy must deal with the resistance that students feel toward speaking with one another. It would be helpful to convince the students that there are benefits to the extra work and exposure to critique. Students could be convinced that they will need to practice social skills in order to improve their workplace-readiness. They could also be convinced that group work or peer review can improve the quality of their understanding and their final marks. It may be easy to demonstrate that interaction with others helps to avoid the more obvious unthinking mistakes. This might be done by assigning homework to be completed in, or discussed by, small groups. This model stands in contrast with the more fully disaggregated model used in online Q&A systems. Incentives in the disaggregated model tend to produce Zipf-like distributions: in an instructional context, it's important that the greater portion of any "long tail" pass above a minimum threshold of quality. Jamie has previously tried using a standard Q&A system for course support. The small-group model asks for a different set of social skills.

> *"Students are not comfortable sharing their work with other students. They are only somewhat comfortable showing it to their professor, whom they regard as one might a medical doctor."* [C8]

Changing content or connectivity / Study and build nonlinear interfaces It may take months or years to get a given model "*loaded into your head*," but once you do, you can then see how manipulating it in one way "*over here*" will produce a change "*over there*." First and foremost, a mathematician looks for tools that will help them build effective mental models. This often means creating useful high-level abstractions. This is different from quasiformality, since the aim is to provide an intuitive, sharable, handle on a topic. For example, a picture is an abstraction that can help a reader to instantly grasp the notion of a Cauchy sequence. If some formalism is used, something very light-weight – e.g. the structured proofs of Lamport (1995) – is preferable to something heavy – e.g. Hales's (2006) Flyspeck.† Parallel points apply to user interface design. As already mentioned, being able to edit a Wikipedia page without logging in is seen as an asset; being able

> *"Formalisms are only useful when the model can't be held in your head."* [C9]

*John McLear, personal communication.

†HTTP://CODE.GOOGLE.COM/P/FLYSPECK/

to quickly click "Was this post useful to you?" helps create a digital trace that might be re-used in various ways; and even a dynamic layout on a web page tends to look "*more professional*" than a fixed-width layout. Related points apply to the technological accouterments of peer pedagogies. Students will not be interested in procedures, like using a course wiki or course forum that do not provide a clear and tangible benefit. Jamie, and others at his institution, seem to agree that the essential skill is "*thinking carefully and critically*" [§4.2, p. 49].

"Wikipedia seems to have better coverage, better exposition, and a nicer-looking interface." [C10]

Changing objectives / Limit philosophizing Of the three implied goals, "quality of exposition" seems like the hardest one to deal with, as it may require rethinking the entire way in which editing and feedback works on PlanetMath. But implementing a *standard* for quality is perhaps not so complex. On Wikipedia, templates are used to indicate that an article "*needs attention from an expert*" or that it "*may not meet Wikipedia's general notability guideline*." We could implement a set of templates for quality in mathematics exposition, so it would be easy to make it clear that a given article "*needs an example*," or "*needs a more intuitive presentation*." Even if without allowing edits from non-logged in users, it would be relatively straightforward set things up so that users could tag articles using a constrained vocabulary.

7.4 A reflective perspective

"Empirical studies of mathematical discourse are a key research focus." [C11]

Changing the nature of the space / Develop empirical studies and a critical apparatus In Alison's research for her PhD (Pease, 2007), she implemented Lakatos's theory of "*informal, quasi-empirical*" mathematics that grows "*through the incessant improvement of guesses by speculation and criticism*" (Lakatos, 1976) in a collection of intelligent computational agents, testing and extending the approach. She also finds significant value in computer mediated communication between humans. Transcripts of contemporary mathematical exchanges provide a more detailed picture of what happens when people do mathematics together than what is found in Lakatos's dialogues, or historical records of mathematical exchanges in old letters. As she put it, this is as close as she can get to having access to mathematician's brains. In "Seventy Four Minutes of Mathematics" (Pease & Martin, 2012), user comments are coded as *concepts*, *examples*, *conjectures*, *statements related to proof*, and "*other*" (an apparatus inherited from Lakatos): Table 14 reproduces the exemplars with which these categories are introduced in this paper. In "informal logic," the focus is usually on proof, but other broad topics of interest for Alison are listed in Table

CONCEPTS - "*Since the points are in general position you could define the* ***wheel*** *of* p*,* $w(p)$*, to be a radial sequence of all the other points* $q \neq p$ *around* p*.*"

EXAMPLES - "*If the points form a convex polygon, it is easy.*"

CONJECTURES - "*Is it possible to prove that any point and any line will do?*"

PROOF - "*Maybe the strategy should be to take out the convex hull of S from consideration; follow it up by induction on removing successive convex hulls.*"

OTHER - linking one thread to another, certifying the proof as complete, etc.

Table 14: *"Concepts, examples, conjectures, proof, and other" - five categories used by Alison Pease and Ursula Martin in "Seventy four minutes of mathematics: An analysis of the third Mini-Polymath project"*

EXPLANATION - "*This seems to be right, but there something I don't understand. Please see if you can help me with it*[.]" [...] "*If I understand well your example: the problem is that you must give an orientation to the line. Then, left and right are define with respect to this orientation*[.]" [...] "*Got it! Kind of like a turn number in topology. Thanks!*" (Pease & Martin, 2012)

UNDERSTANDING - But how does understanding relate to explanation? E.g. often explaining something in greater detail will confuse the listener.

MOTIVATION - Which can be considered in a sociological sense; for example when and where are people motivated by "addictions"?

LINGUISTICS - Including linguistic moves that help to build community, like talking about "*our*" instead of "*my*", etc.

Table 15: *"Explanation, understanding, motivation, and linguistics" - topics of interest in Alison's research*

15. This work is focused on studying what takes place in existing systems rather than on system design. However, platforms that more explicitly represent the kinds of things Alison and her collaborators find in the content would make certain aspects of her job easier; *cf.* (Martin & Pease, 2012).* Lange (2007) describes work on a semantic wiki, and Lange, Bojārs, Groza, Breslin, and Handschuh (2008), an ontology for argumentation that could likely be adapted for this purpose.

*Although they are not system building in the software sense, note the phatic, channel-creating, role of the "linguistic" moves mentioned in Table 15; *cf.* (Elyachar, 2010).

Changing what I know about myself / Find companions for the journey Data from PlanetMath could be coded at the text level or at the hypertext level, and modeled as a forest of interconnected concepts, examples, conjectures, and proof steps. At the hypertext level, the grammatical moves described in Table 10 [§5.2, p. 74] could be given "senses" associated with the categories listed in Table 14, for instance:

"How does experience build towards understanding?" [C12]

$$A \xleftarrow{\text{omits concept}} C, \quad X \xleftarrow{\text{for example}} T, \quad A \xleftarrow{\text{suggests that}} P, \quad P \xleftarrow{\text{partially solved}} S_0$$

If PlanetMath hosts interactions that are of interest to researchers, this would be a good sign that it is doing something that is in a certain sense new – even if this is achieved by simulating the features of older models. Anything researchers are able to learn about the way mathematical discourse works can inform future work on the system, and help to boost the relevance for typical end users.

Changing my perspective / Work with real users An example from one of Alison's earlier studies was the term *landmine*, introduced in a combinatorial problem solving context. This term almost begs to be made into a neologism once again, to describe the knock-on effect of a given new usage. We could measure the impact of a given post (or term) by measuring the number of follow-up posts (or uses). Similar features are modeled in the Behavior Ontology described in Angeletou et al. (2011), using the property `oubo:PostImpact`. This metric could be used to promote important posts to a place where newcomers would easily spot them. A focus on terms means that a forum conversation can be straightforwardly modeled as a graph that is covered by the trees which form its threads.* Graph theoretic techniques could be applied to compute and show the *centrality* of a given person, post, term; or example, conjecture, *etc.*, if this has been formally coded.

"It would be useful to be able to detect the first use of a word within a given episode of discourse, and to measure it's impact." [C13]

Changing content or connectivity / Study and build nonlinear interfaces As discussed above, quasiformality can be difficult for a reader who often relies on informal or non-mathematical language, or pictures, to gain a sense of understanding. One concrete application of network structures within the PlanetMath encyclopedia would be to show the distance of the terms used in a given exposition from a set of basic definitions which are taken as axioms. This would essentially be a topographical map, and gradients and

"Can we build more detailed models of mathematical discourse?" [C14]

* *Cf.* Eco [§4.5, p. 61].

flows could be computed. Such a model could help explain the subjective experience of expository quality in terms of linguistic features (e.g. anaphora) and cognitive models (e.g. cognitive load). With this as a basis, we could then take further steps to explore how knowledge grows and spreads. A graphical approach could be used by research mathematicians to help manage posts and metadata. The user could browse through topics as bubbles, "*open them up*" to zoom in on details of interest, "*attach*" a notebook to a given topic at a given point, or "*connect*" topics with additional comments. This would give a graphical interpretation to maneuvers that currently take place in textual and hypertextual form. A promising related development in the cartography of mathematics is described by Dörrie and Kohlhase (2013), following earlier work by Dave Rusin.*

Changing objectives / Limit philosophizing

"Even if computer mathematics did a good job of simulating human mathematics, it won't do things the same way humans do." [C15]

For comparison, even though there are computers that are very good at chess, they generally do not play like humans. Timothy Gowers and Mohan Ganesalingam (2012) have been working to teaching a computer how to do "*routine mathematical problems*" – that is, problems where "*the obvious thing works*." This, and most other research in computer mathematics, is more or less distinct from research on informal logic. Typical theories of argumentation also focus on proof and truth (Pease & Aberdein, 2011), leaving issues associated with expository quality and collaboration to one side. Appendix B reworks two examples presented by Gowers and Ganesalingam, aiming to illustrate how social theories focused on learning and collaboration might be applied to solve standard mathematics problems. PlanetMath may be able to offer a hybrid digital/analog tutoring system long before a comparable fully computerized system exists that can perform at the same level. Indeed, in contrast to computer chess, it is possible that the relevant target for computer mathematics is not "human mathematics" at all, but social mathematics. "*We may compare a man in the process of computing a real number to a machine*" (Turing, 1936) – but "*the program is best conceived of as a little trial court, or as an evidence-collecting and evidence-weighing procedure*" (Minsky, 1967).

> [W]*hen we write a large program, with many such courts, each capable if necessary of calling upon others for help, it becomes meaningless to think of the program as a "sequence."* [...] [O]*nce past the beginner level, programmers* [...] *write for the individuals of little societies or processes. For try as we may, we rarely can fully envision, in advance, all the details of their interactions. For that, after all, is why we need computers.* (Minsky, 1967)

*HTTP://WWW.MATH-ATLAS.ORG/

7.5 Summary

Table 16 summarizes the findings from qualitative fieldwork with potential users and stakeholders. It illustrates how the paragogy framework can be used to draw out interesting and relevant design themes.

These findings can be applied directly to guide the implementation work described in Chapter 8. Given the ambitious nature of some of the points that were raised, it is likely that the issues discussed here will inform the development of PlanetMath/Planetary for time to come.

This chapter incorporates views from three important categories of potential users: students, mathematicians, and researchers who study mathematical practice. Chapter 10 will return to these findings again to compare them with the views of users of the realized system.

Student	Mathematician	Philosopher
α.	β.	γ.
"Trust" and "credit" are two important and potentially related issues for users/contributors.	We need an efficient system for integrating user ideas and comments.	Researchers can study the content or the system in almost interchangeable ways.
δ.	ε.	ζ.
Progressive problem solving and incremental feedback are important for users, particularly new users who are building their identity as an effective problem solver.	Discussion and critique have a salutary effect; deficiencies in exposition may be a result of their absence.	We should be able to build research-ready data flows as long as the system is actually used.
η.	θ.	ι.
People have instrumental, transactional, information needs and noninstrumental, interactional, learning needs.	People need to learn and practice social skills through interaction.	Graph theoretic methods can be used wherever we have explicit semantics or even simple models of terminology use.
κ.	λ.	μ.
Users and potential users have a lot to say about systems: how can we empower them to actually make changes?	Users want to be able to make adaptations that leave traces, and simple customizations that don't.	Visualizations of graphical knowledge structures could be used directly in the UI.
ν.	ξ.	ο.
Social problem solving is suited to some tasks more than others.	Coverage is good to work on, but quality of exposition is harder.	Computer understanding and formal theories of quality and effective social heuristics are good long-term goals.

Table 16: *Overview of desiderata gathered using the paragogy framework*

Part 4
Results and discussion

8 Implementation

PlanetMath's software system is rebuilt along the lines specified by the paragogy model, augmenting the collaborative authoring environment with tools for peer supported problem solving.

8.1 The Planetary system

In 2010, in parallel to the exploratory research described in earlier chapters, the Planetary project launched. One of the project's key milestones was to rebuild and modernize PlanetMath's software. The project also aimed to create a software system that could be deployed and used in settings unconnected with PlanetMath. In order to facilitate this, Planetary uses the popular content management system, Drupal, as its basis, and connects Drupal with other open source components. The primary participants in this project were Michael Kohlhase of Jacobs University, Bremen, several of Kohlhase's students, and the current author.*

With regard to the system's deployment on PlanetMath, the main objectives were to be feature-complete relative to Noösphere, and to add new features relevant for learning mathematics. These new features include the introduction of problem and solution data types, collections for assembling content into interactive structures, along with other user-visible changes like a new syntax-highlighting source code editor and fast in-browser rendering, and new semantic models in the backend.

My role in the project was to lead the development and deployment of the version of the system needed for the PlanetMath use case. This involved integrating contributions

*HTTP://TRAC.MATHWEB.ORG/PLANETARY/WIKI/PEOPLE

	Basic	**Enhanced**
Interaction	Drupal, ACE2/ShareJS editor	+JOBAD
Storage, Access	Drupal, Virtuoso	+TNTBase, Git
Representation	XHTML+RDFa	+OMDoc
Processing	LaTeXML, NNexus, PyRDFa	+JOMDoc, Krextor
Content	LaTeX (*e.g.* PlanetMath, ArXiv)	+sTeX

Table 17: *Components of the Planetary system*

from other members of the team where relevant. A schematic of the system architecture is presented in Table 17, adapted from the earliest system paper describing Planetary (David, Ginev, Kohlhase, & Corneli, 2010).

Further enhancements to the system are being explored in deployments at Jacobs (David, Kohlhase, et al., 2010; Iancu, Jucovschi, Kohlhase, & Wiesing, 2014), integrating earlier work by members of the KWARC research group on knowledge management tools including Krextor (Lange, 2009), TNTBase (Zholudev & Kohlhase, 2009), OMDoc (Kohlhase, 2001), sTeX (Kohlhase, 2006), and JOBAD (Kohlhase, Giceva, Lange, & Zholudev, 2009). In 2011, an early prototype of the system was selected as a finalist in Elsevier's Executable Paper Challenge (Kohlhase et al., 2011). The system is currently in active use, not only as a web portal on PlanetMath, but also as a course management tool for Kohlhase's introductory computer science courses, and on the new "flexiformal" (Kohlhase, 2012b) mathematics website, MathHub.info.*

PlanetMath/Planetary moves in the directions indicated by the survey presented in Chapter 7, and Drupal's extensibility opens the door to further improvement. For example, the use case sketched in Figure 3 [§2.3, p. 31] has been brought considerably closer at the level of knowledge representation, but the system currently lacks the reasoning support that would be needed to finish the job. I collaborated concurrently on another project that added a recommender service to Drupal (Mulholland, Wolff, Zdrahal, Li, & Corneli, 2013), and integration of the corresponding features into Planetary appears to be simply a matter of time.

What is most essential for current purposes is that Planetary can be discussed, studied, and developed further together with PlanetMath users. The renewed attention to system development restores an empirical aspect to PlanetMath: more than "*kit*" for doing math-on-the-web, PlanetMath/Planetary is a "*research device*" for studying mathematics as a cultural system; *cf.* (Rheinberger, 1997, p. 81). This research programme is developed further in Chapter 9, but the system itself constitutes an important research result, and not just a means to an end. PlanetMath/Planetary is the most concrete synthesis of the ideas that have progressively developed in the course of this work.

*http://mathhub.info

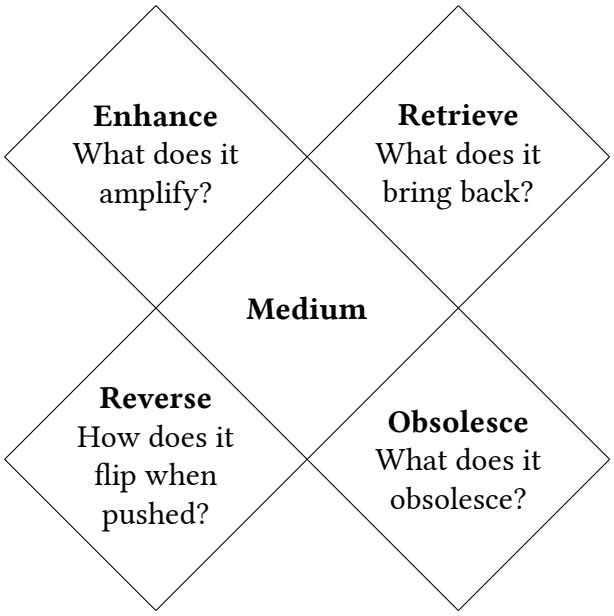


Figure 12: *Marshall McLuhan's tetrad of media effects*

It is a functional actor-oriented model of the way people do mathematics – although it is, of course, only a partial model. The paragogy framework is used below as an organizational device [§3.3, p. 41] to describe the system's high-level specification.

1. **PlanetMath/Planetary should be a special-purpose tool that is customized to suit the challenges of learning mathematics.**
2. **Learning mathematics is an active process that is centered on problem solving; to get the added benefits of peer support, participants have to interact with one another.**
3. **Users will contribute problems and solutions; discussions about problems should spur new engagement among encyclopedia editors and teachers.**
4. **To be viable, the system should facilitate mathematical meaning making; the foundations for this include emergent structure, pragmatic knowledge building, ongoing redesign and evolution at the system level, and the development of new content-focused subcommunities.**
5. **The system should support increasingly refined user and domain modeling, in order to become an increasingly high-fidelity actor-oriented model of mathematical behavior.**

The next section expands these points based on consultations with PlanetMath users and new stakeholders, as described in Chapter 7. After describing the implementation strategy, the following sections present Planetary's new features and analyze them using Marshall McLuhan's (1989) tetrad of media effects (Figure 12).

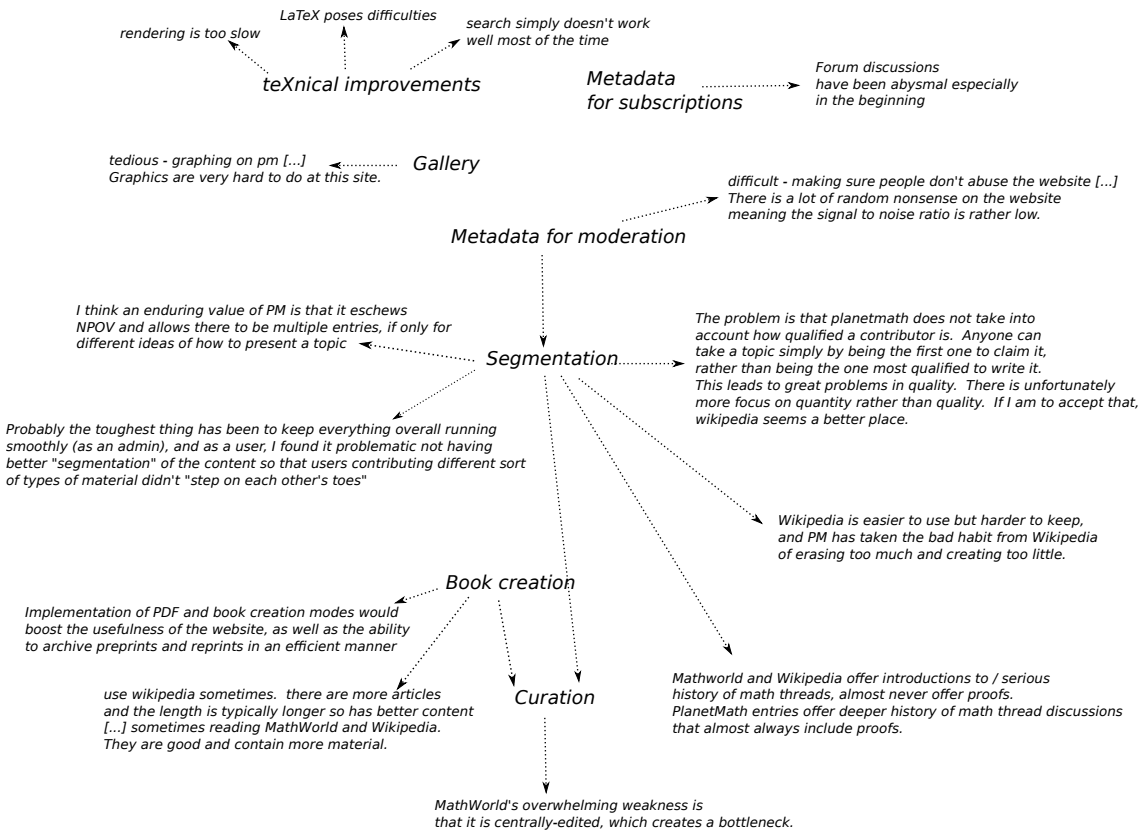


Figure 13: *Participatory design: surveying users of the legacy system*

8.2 Fleshing out the specification

Participatory design

On December 16, 2011, a questionnaire was circulated by email to 30 of PlanetMath's historical top contributors, inviting feedback on an early "alpha" version of the new website as a work-in-progress, which had been deployed on alpha.planetmath.org. The questions aimed to discover the mathematical goals and interests of top contributors, as well as their sentiments about PlanetMath and other related websites, like Wikipedia. Feedback was provided by 10 respondents, and this is summarized in Figure 13. Several central themes are summarized below.

Geometric and visual thinking The frustration people experienced working with images on PlanetMath/Noösphere underscores the importance of visual, analog, thinking for making meaning in mathematics. Improved support for creating and sharing images could only help – particularly for learners who are using the site to build intuition, not just to check the details of a definition.

→ Better support for mathematical diagrams is one feature of Dimension 1 of the specification: the mathematical symbol system includes both analog and digital features.

Different levels of qualification If one were to concede the point that Wikipedia's encyclopedia is a better reference resource – which is convincing enough to be considered – that still leaves the question of a place for everyone who is not yet an expert to engage. In mathematics, these people are often students, and their traditional mode of becoming more mathematically qualified is to solve problems.

→ Adding problem solving support to cater to non-expert users is the essential feature of Dimension 2.

Breadth, Depth, and Quality In-depth treatments of individual topics and broad coverage of many different topics are both considered valuable. Breadth and depth are subject to social and technical constraints, including availability and incentives for experts, and suitable tools for dealing with complex presentations. High-quality exposition and structured presentations are important for anyone learning mathematics. Some people doubt whether a "crowd" can build things like. Better support for making and critiquing books, problem sets would certainly help, along with techniques for semantic modeling that can assist in quality control.

→ One of the key ideas behind Dimension 3 is that feedback from students and careful attention to their needs will help editors find places where exposition could be improved.

Moderation difficulties Related to the point just mentioned, there is tension between having something online *per se* (which people often like, since it potentially helps them gain exposure, get feedback, and find potential collaborators) – and having something in the encyclopedia, where others end up having to interact with it. For example, my own uploads from APM-Ξ didn't fit well in PlanetMath's encyclopedia, but I would have liked to find some place for them on PlanetMath (Chapter 2).

→ Suitable strategies for segmentation would help with many of the sub-points of Dimension 4.

PlanetMath's unique model The ownership model blends "Wikipedia" and "Scholarpedia"* approaches. Raymondite homesteading is part of this model, and the intent is that this will give experts the chance to manage their own content. However, this comes with some of potential quality control issues: what if instead of improving an article, an owner only "squats it"? In fact, users who don't respond to corrections must orphan their articles – but the ownership model still slows down change relative to Wikipedia. An improved co-authoring workflow could take care of some basic points of conflict. In

*HTTP://WWW.SCHOLARPEDIA.ORG

addition to ongoing discussions with users, better data gathering and analysis should help detect places where users are running into difficulty.

→ Understanding and better serving user needs is one of the key applications of Dimension 5.

Connections to stakeholder desiderata

Table 18 further expands the specification by referring it to the desiderata identified in preliminary field work (as summarized in Table 16 [§7.5, p. 104]). This yields a detailed outline of work for the implementation project.

This expanded view shows how the five focal dimensions of change at the system level are, here, projections of a more complicated change process. Rather than "dimensions" it may be more accurate to speak of them now as "subspaces," or "subprocesses," with the further caveat that these are not wholly distinct, but that they will interact with one another. Each of the five subspaces traces a virtual engagement between the dramatis personae Student, Mathematician, and Philosopher, in the (changing) context of the system-under-development. All of these figures are themselves involved in their own processes of change [§3.3, p. 41]. When compared with Chapter 7, what's new here is that it is now seen how these change processes can be related to each other.

The descriptions presented in Table 18 were not obtained by reading across the rows of Table 16, but by refactoring its elements to tell a more coherent story. For example, along Dimension 3, through the hinge-pin of *motivating users to contribute problems and discuss solutions*, trust and credit (α) are now seen as connected with discussion and critique (ε) as well as data-driven research looking at topics like the way understanding develops in dialog (ζ). As the nature of the student's environment changes so that he or she can get credit for participating in activities online, the mathematician and philosopher must increasingly understand their roles to be connected to discourse. The system, correspondingly, should adapt to support participation at varying levels of familiarity and literacy, not only with regard to mathematical concepts and proof techniques, but also the relevant forms of social engagement.

This broad programme still only points in a general way to what needs to be implemented, rather than to a manifest of specific implementation tasks. Geertz notes that the separateness of plans of this type from actual cultural practice "*makes them less useful as models for the interaction of cultural patterns*" (Geertz, 1973, p. 250). On analogy with Vygotsky's view that "*the primitive child is a healthy child*" (Vygotski, 1929), the implementation approach will focus on the minimum kernel of features that is needed to build a concrete working model, rather than an abstract picture. Interacting with a realized system will render the subsequent discussions and critique comparatively concrete.

Dimension 1 (Desiderata η, λ, μ): "A special-purpose tool customized to suit the challenges of learning mathematics." The system must deal with both informational and interactional needs of users. It should allow users to make learning-relevant adaptations and customizations. It should help people visualize and interact with mathematical knowledge – diagrammatic thinking is one basic aspect of this.

Dimension 2 (Desiderata δ, θ, ο): "An active and interactive process centered on problem solving." This should include features of progressive problem solving and incremental feedback, as well as the opportunity to learn and practice collaboration skills. We should build better models of expository quality and effective problem solving heuristics, and the relationship between them. As students become better at solving problems, they become more mathematically qualified: ideally, the system should be useful wherever the user is in their mathematical literacy lifecycle.

Dimension 3 (Desiderata α, ε, ζ): "Motivate users to contribute problems and discuss solutions." In order to accomplish this, we will have to address the issues of *trust* and *credit*. If we want to motivate new levels of engagement from editors and teachers, we will have to highlight the efficacy of discussion and critique. We should identify the meta-mathematical research problems associated with discourse in the space, and help researchers work on these problems. Following Timothy Gowers, let's ask: "Is massively collaborative mathematics instruction possible?"

Dimension 4 (Desiderata β, γ, κ): "Support mathematical meaning making within the system and through ongoing system development." Will user comments be heard and acted upon (whether at the level of "system" or "content")? Can users themselves contribute to development? Can we develop a rigorous approach to system change and development? One concrete problem is the tension between making it easy for contributors to add material, and making the things the reader sees high-quality. This invites an innovative approach to segmenting the site.

Dimension 5 (Desiderata ι, ν, ξ): "Support user and domain modeling." Can we discriminate between the tasks where social problem solving works better than individual effort, and vice versa? Can we understand the behavioral and semantic features of high-quality exposition? How can we facilitate graph theoretic analyses of questions like these? How can we understand the impact of system features (like the ownership model) on the user experience?

Table 18: *Design dimensions*

8.3 Implementation strategy

Figure 14 connects the specification described in the previous section to concrete system features. As can be seen by comparing with Figure 13, many long-standing feature requests have been addressed in the process of this conversion as well.

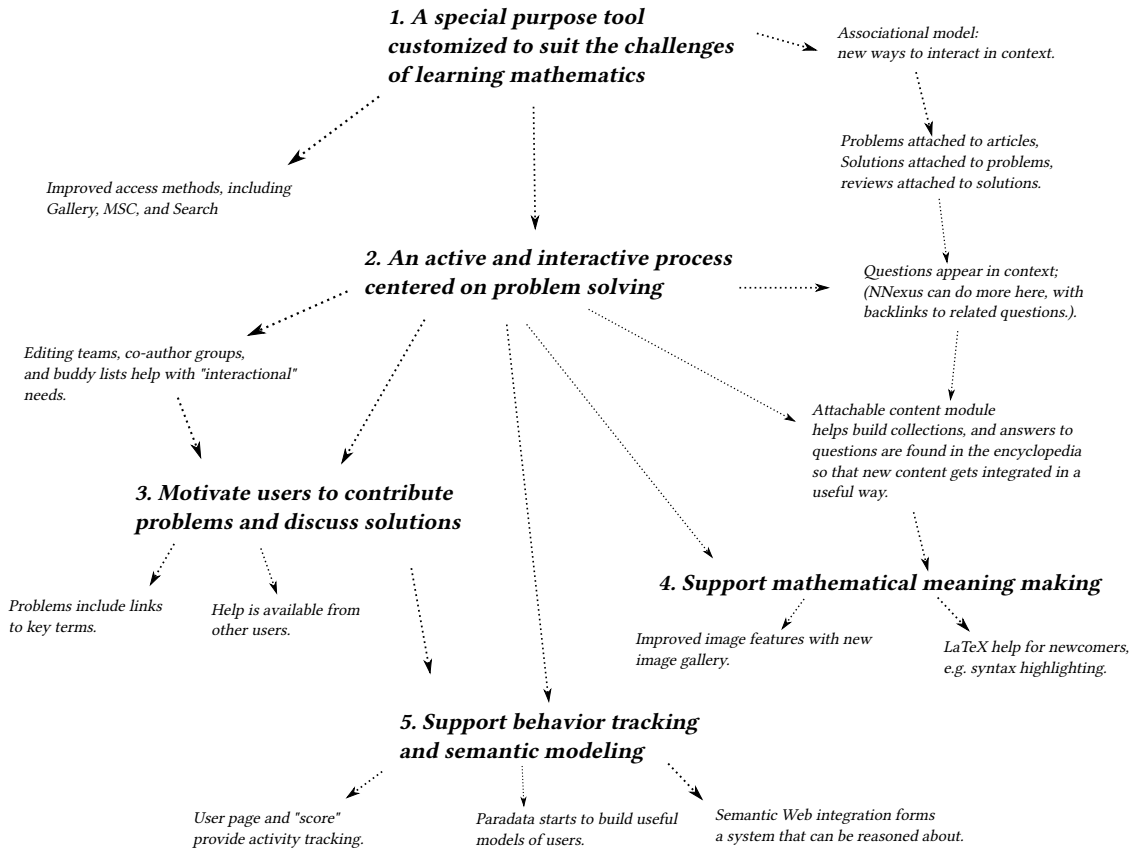


Figure 14: *Connecting the design specification to concrete system features.*

In order to realize this design and develop a solid foundation for further work, development began by rebuilding the main features of Noösphere from the ground up – using the extensible content management system (CMS), Drupal 7.* This allowed us to modernize and modularize the system in the process of building our enhancements. It cannot be emphasized enough how much an easy to install system built as customizations to a mainstream CMS helps with extensibility when compared with a difficult to install custom codebase.† This is important in light of the fact that we could not realistically do everything that everyone would have like to see done in the time available. An extensible modern system that combines feature-parity with Noösphere and new problem solving features was sufficient for current purposes. Not only is the system immediately usable

*HTTP://DRUPAL.ORG/DRUPAL-7.0

†Full installation instructions for Planetary are available on Github: HTTPS://GITHUB.COM/KWARC/PLANETARY/WIKI/INSTALLATION-INSTRUCTIONS

– featuring some noticeable improvements over Noösphere – it also allowed collaborating developers to build new working prototypes that open up interesting directions for further work. Development took place on Github to facilitate distributed revision, and developers made use of the integrated issue tracker.

In addition to Drupal, two other core components come together to form the new Planetary system. These are: (1) LaTeXML,* which we used to provide a modern, fast rendering system that supports semantic markup; and (2) the Virtuoso triple store,† which allowed us to begin to make use of semantic markup in a computational manner.

When compared with Noösphere, Planetary is better able to expose semantics at the fine-grained textual and bulky hypertextual levels. This means that in the future, much more information will be available for analysis along the lines indicated in Chapter 6.

As is the nature of open source collaboration, Planetary builds on a considerable body of prior work; *cf.* (Kohlhase et al., 2011). PlanetMath is the first major public demo of the Planetary platform. After a short overview of the system as a whole, this chapter will focus on *problem solving support* (problems, solutions, reviews, collections, questions), *data management* (personalized tracking, SPARQL integration via Virtuoso and PyRDFa), and *support for groups*, which I've been primarily responsible for developing in order to build PlanetMath/Planetary. Details on other features of the Planetary system – including real-time collaborative editing, Javascript-based interaction for readers, mathematics-aware search, and expressive semantic markup via sTeX – can be found in papers by other members of the Planetary team (Jucovschi, 2012; Kohlhase, 2012a; Kohlhase et al., 2009; Kohlhase, Matican, & Prodescu, 2012; Kohlhase, 2006, 2008).

Overview of Planetary's features

After a first round of prototyping with Vanilla Forums‡ (Kohlhase et al., 2011), we settled on using Drupal 7 as the basis for the system. Drupal is a popular open source content management system, known for its modular approach. Many contributed free/open source modules are available, and the Drupal developer community offers a range of professional services. For processing mathematical text, we used LaTeXML, a parser that takes in LaTeX source code and generates XML, and from that, HTML+MathML.§

> *MathML is a low-level specification for describing mathematics as a basis for machine to machine communication which provides a much needed foundation for the inclusion of mathematical expressions in Web pages.* – http://www.w3.org/Math/

*http://dlmf.nist.gov/LaTeXML/
†http://virtuoso.openlinksw.com/dataspace/dav/wiki/Main/
‡http://vanillaforums.org/
§http://dlmf.nist.gov/LaTeXML/

From the user perspective, a significant advantage of MathML is that it transfers and renders much faster than other alternatives. It is rendered natively by an increasing number of web browsers, and the Javascript-based MathJaX provides a fallback solution in browsers where native MathML rendering is not available. MathML also lays the foundation for interactive in-browser services (Kohlhase et al., 2009).

We were able to use several contributed Drupal modules with zero configuration to reproduce some of the old features from Noösphere. Examples include the contributed privatemsg and watcher modules. Other modules could be used with some minor customization (*e.g.* scorekeeping via userpoints) or integration via an API (*e.g.* groups support using the Organic Groups module, og). Some features naturally had to be custom-built, for example, a new corrections module, and the drutexml module, which communicates with LaTeXML running as a web service.

Constantin Jucovschi used the wysiwyg module to add a custom real-time collaborative editor, based on ShareJS.* He is extending the editor with "bots" that provide a range of in-editor services (Jucovschi, 2012). The only code remaining from the legacy PlanetMath codebase is the NNexus auto-linking module, which Deyan Ginev has refactored into object oriented Perl, and adapted to run as a web service along the lines of the LaTeXML web daemon, which he also maintains; *cf.* (Ginev & Corneli, 2014).†

Problems and *solutions* are the key new objects, along with *questions*, *collections*, and *groups*. Working with problems and solutions, and integrating them with expository material from the encyclopedia requires new workflow and metadata management features. Table 10 [§5.2, p. 74] summarized the main activities planned for the new system, and connected these activities with the five dimensions of paragogy. Figure 14 now provides a significantly expanded view of a paragogical design for mathematics learning.

Many of the new features are familiar from other wikis, online communities, or offline problem solving contexts. Collections do require a brief introduction. The available interactions are similar to those associated with YouTube playlists,‡ *i.e.* collections are designed to help users organize articles, problems, and other content. In the mathematics context, collections are useful for sharing reading lists, problem sets, and more complex curricular objects; *cf.* [§1.2, p. 6]. A similar feature was developed by Silvan Reinhold (2006) for the WikiTrails system, which was designed to augment a MediaWiki installation with navigational and learning pathways. Trails could be created in "*automatic, semi-automatic, as well as manual modes.*" Christoph Lange (2007) points out that the knowledge embedded in wiki pages is not used by the WikiTrails system. In the current prototype, the collections mechanism integrates information about user interactions and

*https://github.com/jucovschi/ShareJS
†https://github.com/dginev/nnexus
‡http://www.youtube.com/yt/playbook/playlists.html

the hypertextual context. Deeper semantic integration is left for future work.

In the following section, the system features that were presented in capsule form in Figure 14 are unpacked and explored with screenshots from the live system deployed on PlanetMath.

8.4 System walk-through

Figures 15–25 show some of the key features of the developed system. The pages can also be browsed on PlanetMath.* The screenshots and discussion are organized in terms of the five dimensions of the design described in Table 18.

The clusters of features associated with each of these dimensions is reviewed in terms of the McLuhan tetrad (Figure 12), which helps to focus on specific, localized changes, in contrast, for example, to the more global concerns addressed by Richardson and Domingos (2003) [§4.5, p. 61]. Some of the pragmatic features associated with building large knowledge bases are enhanced by the new features and where something can be said about this it will noted – but there are other cultural factors that are relevant to peer produced peer learning that McLuhan's formulation is better at capturing.

1. A special purpose tool customized to suit the challenges of learning mathematics

Figure 15 illustrates the first point where a user might encounter one of the key new features of the system. When browsing an article, a block containing links to attached problems appears, if the article has attached problems. If there are no attached problems, the reader is invited to add one. The figure illustrates additional interactions that are available to any logged-in reader, when browsing an encyclopedia article. The reader can add a correction, ask a question, or add the article to a collection. Both the text of the article and the "look and feel" are worth commenting on.

In the past, PlanetMath used a range of different rendering systems. The earliest of these was LaTeX2HTML,† a system that converts mathematical expressions into in-line images. The most recent system used by Noösphere was jsMath, the immediate predecessor to the more widely-adopted MathJaX. Javascript-based client-side rendering with these tools is considerably slower than native rendering of MathML within the browser, which is currently best supported by Firefox's C++-based Gecko engine. Native support for MathML is not present in all modern browsers, so Planetary includes MathJaX as a fallback option. LaTeX source code and rendered PDFs are also made available.

*Up-to-date screenshots, together with links to live pages on the system, are collected at HTTP://PLANETMATH.ORG/PLANETARYRELEASENOTES.

†HTTP://WWW.LATEX2HTML.ORG/

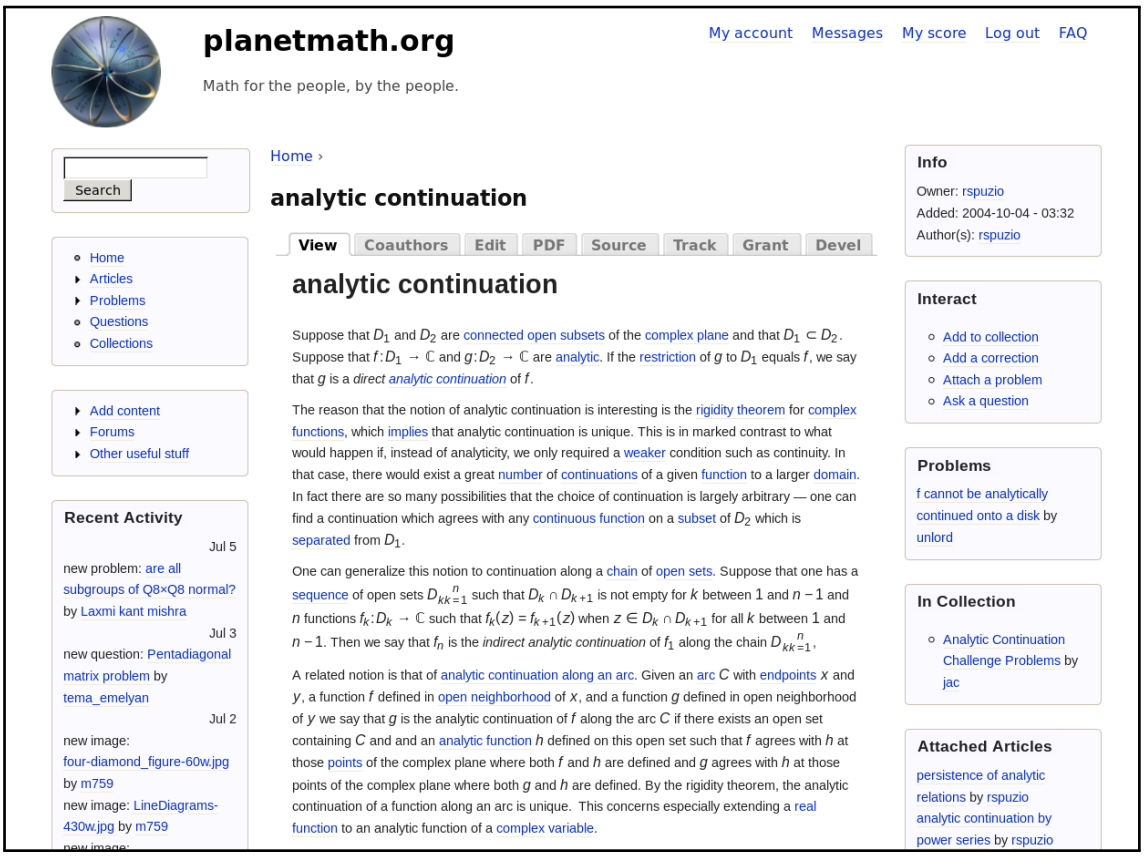


Figure 15: *"Article UI" (reader view), including problems attached to articles.*

The three-column theme was implemented by Alex Dumitru using Drupal's popular zen theming system (version 7.x-3.1).* The idea was to provide the basic interactions and site messages in a simple version of a "*heads-up display*".† One designer I spoke with was not greatly impressed, and said that the site has an "*early nineties look.*"‡ As noted in Chapter 7, a layout that used all of the space on the page for text might look "*more professional*" [§7.3, p. 99]. Some additional theming and Javascript work would enable us to make the sidebars stay hidden unless needed, but at the outset of the project there were other more pressing issues to solve.

The HTML produced by LaTeXML expresses mathematical symbols using their Unicode representations. Some of the "obscure" symbols (like Fraktur fonts) occupy registers beyond the Basic Multilingual Plane (0x0000 - 0xFFFF) supported by MySQL's UTF8 encoding. Planetary seems to be the first system to use LaTeXML together with a MySQL database. A resolution was reached by upgrading MySQL and converting to the richer UTF8MB4 encoding,§ and creating a (local) patch for Drupal's core database module.

*HTTPS://DRUPAL.ORG/PROJECT/ZEN
†HTTP://EN.WIKIPEDIA.ORG/WIKI/HEAD-UP_DISPLAY
‡Anna Keune, personal communication
§HTTP://DEV.MYSQL.COM/DOC/REFMAN/5.5/EN/CHARSET-UNICODE-UTF8MB4.HTML

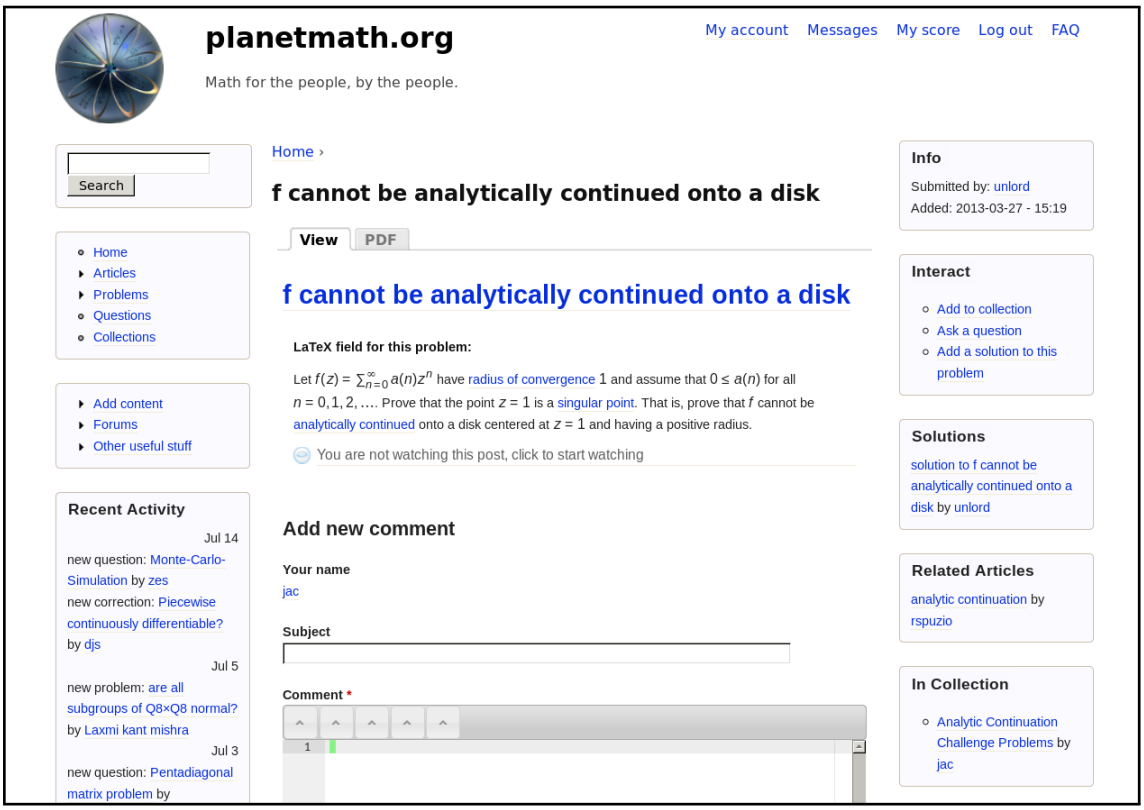


Figure 16: *Problems are attached to articles; solutions are attached to problems*

Figure 16 shows a problem, with its own set of interactions, including the ability to add a new solution. Once solutions have been added, they will be listed in another block. Problems also come with a "Related Articles" block that shows the articles to which the problem has been attached (if any). Some users might prefer *not* to know that solutions are available until they specifically ask for a hint; Drupal would make it easy to accommodate this sort of user preference.* In terms of our ability to control system features at this level, we can compare Planetary with WorkingWiki, a MediaWiki extension for interacting with source code of various forms (including LaTeX).† WorkingWiki is an excellent program that pushes the envelope of what can be done with wikis – and, like Planetary, it uses LaTeXML for rendering. The ease with which different kinds of content can be created and connected together in Drupal confirms that using a CMS is a good choice for the Planetary project. Interconnections between different kinds of content, and interactions around these connections are the essential features of the design (Figure 1 [§1.4, p. 12]). Using LaTeXML opens the door to interaction with individual pieces of content, via "deep links", since each element on the page is supplied with a unique identifier (`xml:id`).‡,§

*https://github.com/KWARC/planetary/issues/343

†http://www.mediawiki.org/wiki/Extension:WorkingWiki

‡http://lists.jacobs-university.de/pipermail/project-latexml/2013-April/001291.html

§https://groups.google.com/forum/#!topic/planetary-dev/xP0PiSQVQNE

Enhance The improvements described here target the relational features of the PlanetMath system. Not only are objects related to other objects in new ways, the text itself is enhanced, which facilitates new ways for people to interact with system contents. For instance, it is now possible to link to and discuss individual elements of the page. At both "micro" and "macro" level, the system has better support for user interactions.

Obsolesce The first target of the Planetary project has been to obsolesce Noösphere by building a system with feature parity. A potential longer-term goal is to obsolesce other online or offline mathematics resources, by growing to include feature parity with more of them. At this stage, the key point is to support customizability and extensibility in Planetary system at all levels.

Retrieve Reference material is now connected explicitly to a problem solving context. The sense of mathematics as a subject that is only learned actively is brought into focus in the new system.

Reverse When the relations between different topics are developed further, it may be easier to conceptualize PlanetMath's content not simply as a textual or hypertextual corpus, but as a graphical image or interactive mathematical object. The MathMap project by Jan Wilken Dörrie at KWARC offers an early prototype for this experience, and currently provides an alternative access method to PlanetMath's content.*

2. An active and interactive process centered on problem solving

Figure 17 shows the new LaTeX editor in action, with the user in the process of adding a solution to the problem that was shown in Figure 16. The text of the problem statement has been automatically retrieved and copied onto the screen above the LaTeX field, so the user doesn't have to open multiple tabs to keep track of the basic information they need to solve the problem. This is achieved by using a custom *view mode* and a custom template for rendering the solution entry form. The new '`content`' view mode produces only the content of the problem, without the perpheral features that appear when the problem is rendered as a webpage.

By following the links to technical terms used in the problem (here, to PlanetMath's articles on "*radius of convergence*", "*analytic set*", and "*analytical continuation*"), the user can remind him or herself of the definitions. In the future, we could do much better, by generating a "course packet" on the fly to assist the problem solver. The latest version of NNexus embeds RDFa into links, which makes the connections between pages

*HTTP://MAP.MATHWEB.ORG/

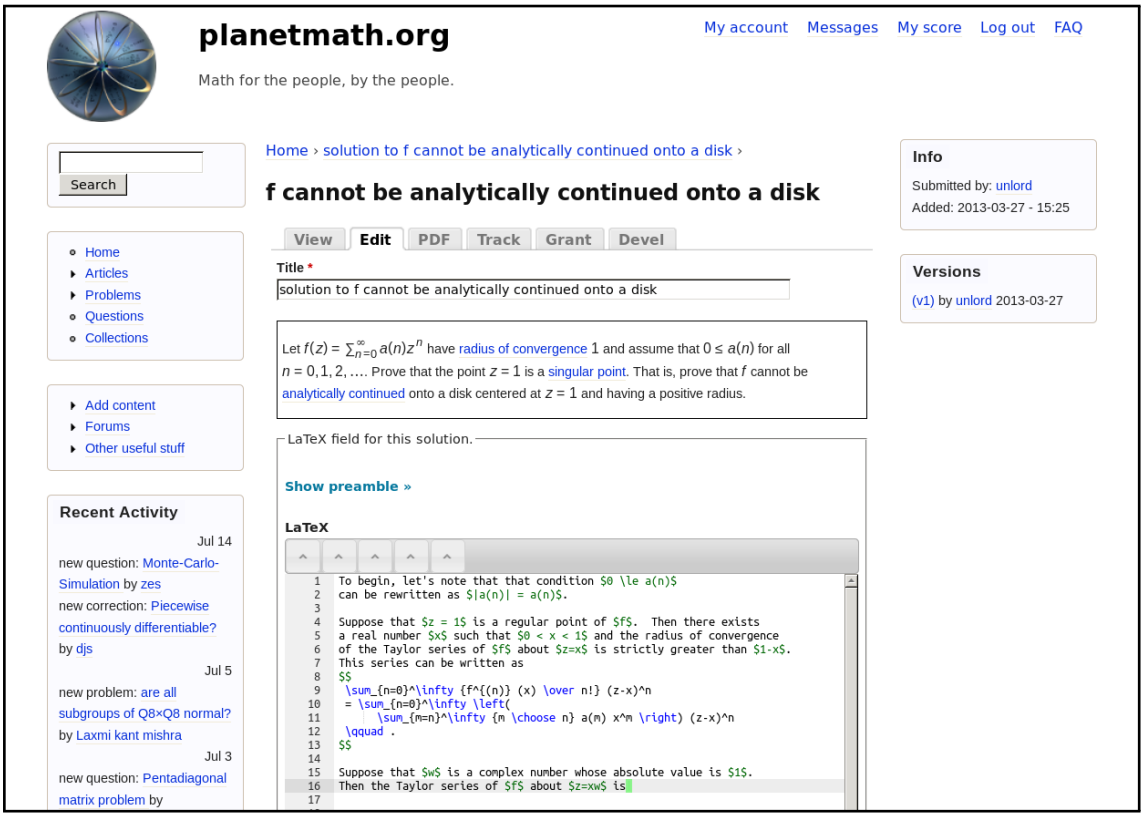


Figure 17: *Solving a problem*

explicit; the relevant information for building this sort of on-the-fly course packet is now available via a SPARQL query.* However, to deliver good results, it will be important to develop better strategies for NNexus to use to steer links accurately, since there are often different definitions for the same term: some of these strategies include steering based on the Mathematics Subject Classification, entropy-based approaches, and adding mechanisms for users to give feedback on mis-directed links.† Improving NNexus's accuracy is currently a work in progress (Ginev & Corneli, 2014). In the event that a problem is still too hard to solve, the user can ask a question via one of the interactions offered in Figure 16. This is in line with the example use case for combined peer learning and peer production workflows that was described in Figure 3 [§2.3, p. 31]. Links to easier (or harder) related problems would be useful to add, and with more metadata (for instance, the "mathematical depth" of the terms used in the problem [§6.4, p. 88]), these too would be a SPARQL query away.

Although automatically-created reading lists and problem sets are just beyond the Planetary system's current capabilities, we have stable support for building them manually, via the collections mechanism.

*https://github.com/KWARC/planetary/issues/323
†https://github.com/dginev/nnexus/issues/34

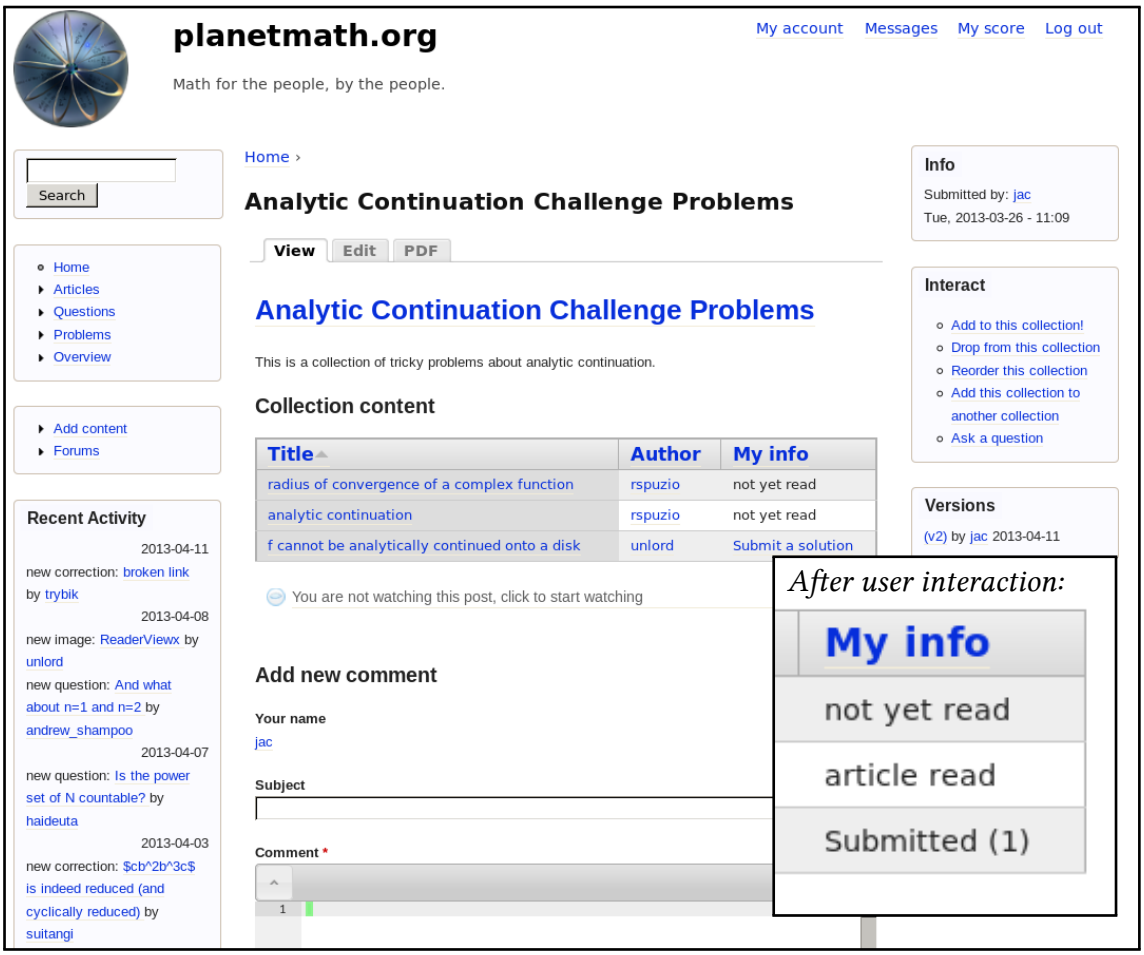


Figure 18: *Paradata that shows up as a user works their way through a collection*

Articles, problems, and other content can be added to collections, which function as "playlists" for math content. As illustrated in Figure 18, when the user works their way through the problems in a given collection, personalized metadata (or, more precisely, *paradata* [§4.4, p. 57]) shows up in a field labeled "My info". If another user adds a review for a contributed solution, this information will appear as well.

Although collections are just ordered lists, they can also contain other collections, allowing the user to create and share a tree-like outline, or more general network structure of paths through the content. There are not yet any specific UI features for managing sub-collections in PlanetMath, but our early prototype included "Mathematica-like" folding and unfolding interactions, to show and hide various elements on the page (Kohlhase et al., 2011). This would make the user's interaction with complex structures built using the (sub-)collections facility more intuitive.

An "expand all" feature is the obvious next step, since this would give users an experience closer to reading familiar PDF documents. This could be implemented using the content view mode described in connection with Figure 17.

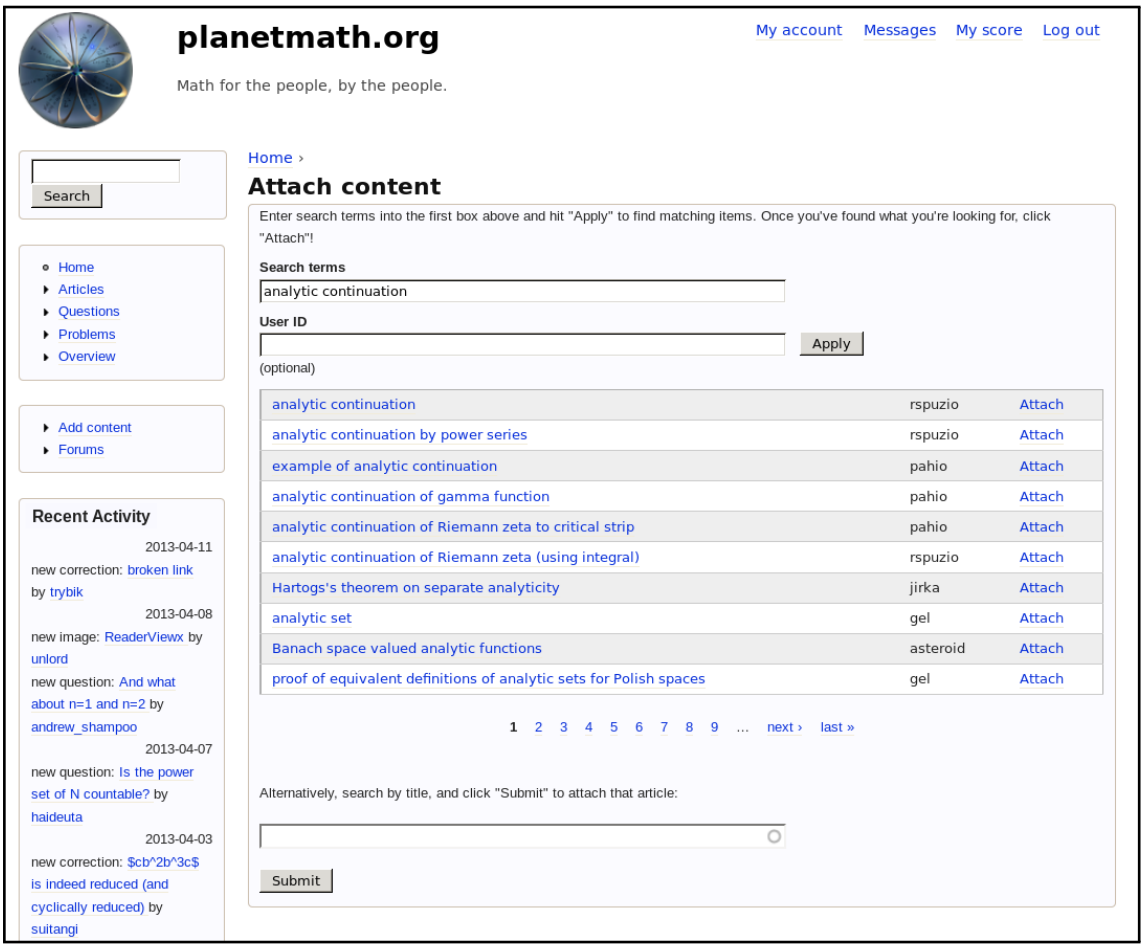


Figure 19: *Search being used to add items to a collection*

Figure 19 shows a common UI that is used for building collections, adding content to groups, and also for attaching answers to questions. These interactions were developed in the planetmath_attachable_content module. They make use of dynamic Drupal views that are built using free-text search, which is provided by Drupal's integration with Apache Solr and the associated apachesolr_views module.* Only minor changes were required to index mathematics documents properly,† but integration with a truly mathematics-aware search engine, like MathWebSearch (Kohlhase et al., 2012) remains for future work.

New objects can also be added to a collection by autocompleting on the title, if the title is known – or added *in situ* while browsing by using the link in the "Interact" box (indicated in Figures 15 and 16). New localized interactions are easy to add: so far, there are custom interactions defined for articles, groups, problems, solutions, questions, and collections. The "Interact" box can display different contents for authors/owners and other users; at the moment it does not display at all for non-logged-in users.

*HTTPS://DRUPAL.ORG/PROJECT/APACHESOLR_VIEWS
†HTTPS://DRUPAL.ORG/NODE/1827260

On a whole, the problem solving support is designed with these core features in mind:

- *A low floor*: keywords will be automatically linked to their definitions, which should help users get past a "cold start"; it is also easy to participate just by asking a question; and,
- *A high ceiling*: the collections feature can be used to assemble problem sets or books; users can explore advanced topics, and help others;
- *Support for independent study*: Built-in activity tracking conveys a sense of personal progress, and an increasingly rich collection of metadata and paradata will be useful for recommendations;
- *Support for teachers*: Teachers will be able to use the system to run their own courses – compare earlier classroom experiments run by David Smith (2002) and Robert Milson (2005); Planetary can be used either "as a service" on PlanetMath, or downloaded and further customized in a local installation elsewhere.

Enhance The new problem solving features in PlanetMath/Planetary develop richer dynamic relationships between objects and users by adding the ability to build structure on top of user-contributed materials, and by modeling learning progress.

Obsolesce The collections feature is intended to obsolesce paper-based problem sets. However, with some relatively minor enhancements, it could also make *textbooks* obsolete. PlanetMath content is free and sharable, so it can clearly undercut other offerings in price. Furthermore, unlike textbooks, PlanetMath comes with peer interaction built in.

Retrieve Whereas a hypertext encyclopedia tends to be labyrinthine [§4.5, p. 61], collections can help make the progressive dimension of learning more explicit. They restore the sense of linear order that is familiar from problem sets, textbooks, and state mathematics curricula.

Reverse As PlanetMath builds a more complete catalog of problems, collections, and solutions, we accumulate a machine-readable archive of solution strategies and learning pathways. This archive could potentially be used by artificial agents working on mathematics problems. One application of such a system would be to support the development of a knowledge-rich intelligent mathematics tutoring system. Rather than users teaching the system by adding and structuring content, the system would be actively engaged in teaching users.

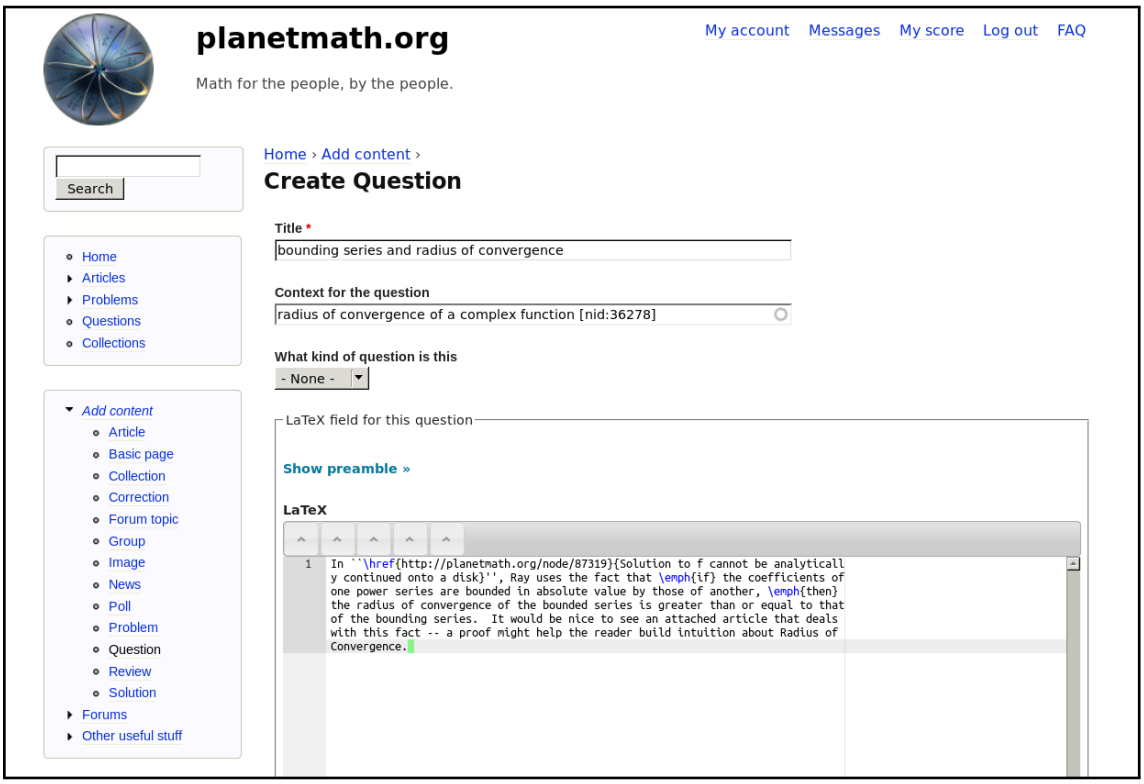


Figure 20: *Adding a question*

3. Motivate users to contribute problems and discuss solutions

Figure 20 shows a question that is being added "in context" – the user would have gotten here by clicking on the "Ask a question" UI element that appears in the sidebar on articles, problems, collections, and solutions. The user can also ask a question without specifying the context by using the "Add content" menu item from the left sidebar directly; in this case, the "Context" field is hidden from the UI and is left blank when the user presses "*save*."

Questions from across the site show up in a special-purpose feed.* This is one place where we deploy a Drupal *view*. Views offer the possibility for relatively straightforward theming. With more work, we would be able to reproduce many of the same features from StackExchange or its existing open source clone, OSQA.†

Some questions will be useful student-level exercises, like the problems discussed above; but many can be thought of as problems of exposition, and will suggest improvements to the encyclopedia. As with other PlanetMath data types, questions are discussable and can have comments attached – however, *answers* are supposed to be presented in encyclopedia articles, as will be illustrated shortly.

*HTTP://PLANETMATH.ORG/QUESTIONS
†HTTP://WWW.OSQA.NET/

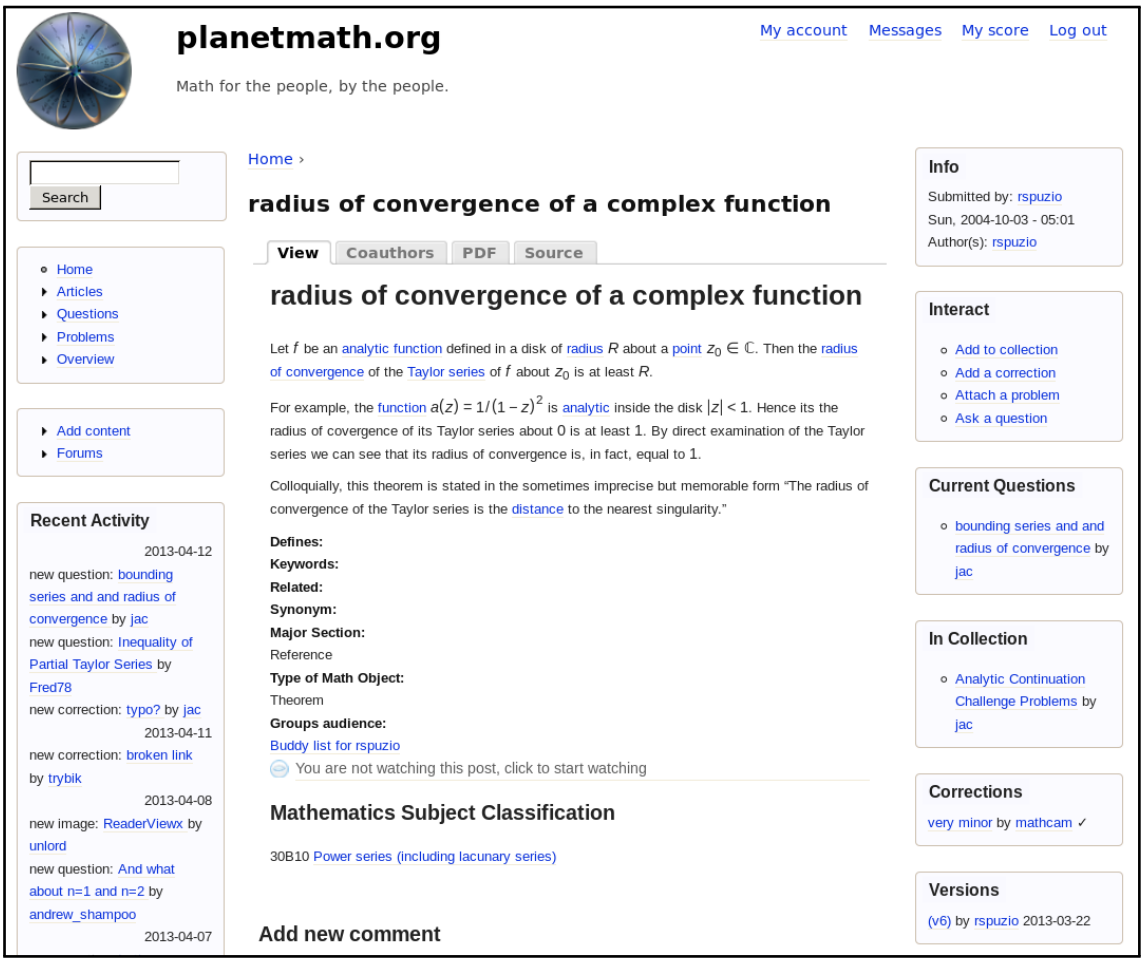


Figure 21: *Current questions appear in context*

As depicted in Figure 21, if a question is asked in context, a link to the question shows up in the right-hand sidebar for everyone browsing the relevant object, as a "Current question." New questions also show up in the left-hand sidebar along with other recent activity, and this is persisted across the site, providing a form of light-weight free advertising for users who contribute questions.

Using more refined notions of context, we could adapt this model, so that recent activity would show up sorted by MSC topic. As we shall see in Chapter 9, there are many reasons to reconsider PlanetMath's current orientation towards only the most-global and most-local contexts.

Although contextual questions serve a similar functional role to that of corrections, they are meant to appeal to a more naive reader. A flow of naive questions may a very good way to find and address flaws in exposition. This is consistent with the view that "*the best way to learn is to teach*" (Luk & Wang, 2012). Any constructive reader interaction should help to enhance quality "at the boundary" of author-controlled articles; *cf.* [§1.2, p. 5].



Figure 22: *Making improvements to the encyclopedia when answering a question*

As mentioned above, in the question answering model deployed on PlanetMath, all answers are supposed to be supplied in encyclopedia articles. When a contributor indicates that they would like to answer a question, they are presented with an interface like the one in Figure 22. This is the interface from the planetmath_attachable_content module, similar to what was shown in Figure 19. Additional explanatory text and a link are presented when the user indicates that they want to answer a question:

> *The way question answering works around here is that answers are found in encyclopedia articles! If you can't find an existing article that answers the question using the tools below, you can* ***create one*** *and it will be automatically attached as the answer when it's saved.*
>
> planetmath_attachable_content module, emphasis indicates hyperlink

The answerer can either point to an existing article, and add a brief memorandum to the reader – or add a new article. In practice, many users seemed to find this aspect of the interaction confusing. Contributed questions were often answered in *comments* attached to the question, rather than by connecting the question to articles. This does not immediately invalidate the idea of using questions to improve the encyclopedia, but does suggest that the workflow will need to be streamlined. As another approach, one that is particularly interesting in light of the fact that a highly-effective workflow related to question answering already exists on sites like math.stackexchange.com and MathOverflow, various integrations with these sites may be worth pursuing. For example, it should be possible to develop machine learning techniques that provide automated or semi-automated means of connecting questions (and answers) to relevant articles.

Enhance With the introduction of an explicit "question" object, texts contributed to the site start to become more explicitly typed. This moves in the direction of the ontologies for argumentation developed by Christoph Lange (Lange et al., 2008); (Lange, 2011, Chapter 3).

Obsolesce Questions may contribute to obsolescing the old corrections mechanism, at least in part. The question-answering mechanism makes the explicit point that unclear aspects of one article may be explained better in another. The question mechanism also directly obsolesces the old "requests" mechanism from PlanetMath/Noösphere – old requests are mapped into the new system as a question sub-type.

Retrieve Unlike other Q&A sites, wiki discussion pages, or the combination of Q&A with wikis, this system couples questions closely with content. This restores a spirit of camaraderie to the process of addressing user concerns, and points in the direction of collaborative R&D instead of simply Q&A. In short, it retrieves a robust sense of "We". (Compare Table 2 [§3.1, p. 35].)

Reverse Once many questions have been asked and answered, there will be less reason to ask questions anew, since the old answers can just be read. If pushed far enough, the question-answering tool will produce a set of very clear expositions, in the form of repeatable instruction sets. More aspects of the mathematical experience would become algorithmatic – comparable, perhaps, to sewing patterns, cooking recipes, or driving directions.

4. Support mathematical meaning making

Two new features of Planetary are particularly geared towards meaning making. A new gallery allows users to share images between articles. Formerly, in Noösphere, each article had its own "file box", to which images could be added. This was useful for adding images to individual articles, but it did not provide a central place for learning about and discussing mathematical images. As noted by legacy users (Figure 13), making these images can be quite a challenging process, often involving custom scripting or programming. The current gallery provides the early beginning of a strategy to address these issues. Groups and collections are intended to address several aspects of meaning making that were identified in the research literature. In short, collections are useful for structuring content, while groups are useful for structuring activities. Planetary includes three kinds of groups: teams, buddy lists, and co-author groups. Groups provide a more intuitive way to control permissions than Noösphere's old UNIX-like Access Control Lists – and they also give a platform on which to continue building features that offer improved support for organizing around shared interests.

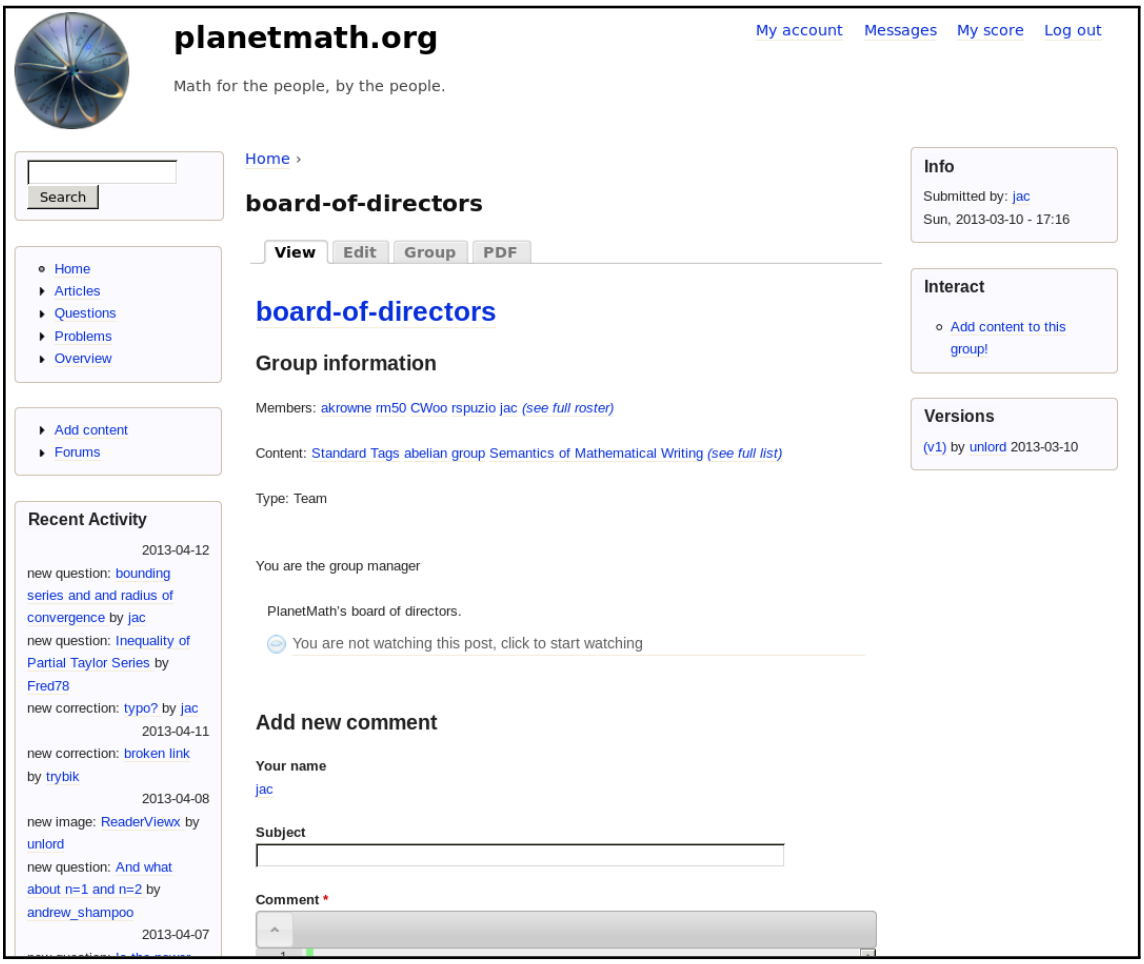


Figure 23: *Groups workflow: a team's shared content*

Figure 23 shows a team's content. Any content that is added to a team is made editable by all of that team's members – content can only be added to a group by someone who already has editing permissions. Buddy lists are similar. The members of an individual's buddy list have write access to all of his or her articles. Optionally, a team can be given access, instead of a list of individuals. In particular, if a user wants to give everyone access to their articles, they can designate the World Writeable team as their buddy list.

Co-author groups are specified on a per article basis. Permission to edit an article granted via one of the mechanisms described above *does not* automatically make you a "coauthor," although all edits are recorded in the article's revision history. This allows trusted members of a buddy list or team to make small corrections, without assuming full co-author status. Groups were implemented using Drupal's Organic Groups module.* Various desirable enhancements (like per-group forums and mailing lists) are available as contributed modules, but are not always well supported. Drupal Commons is a suite of modules, also using og, that could provide further enhancements or inspiration.†

*HTTPS://DRUPAL.ORG/PROJECT/OG
†HTTPS://DRUPAL.ORG/PROJECT/COMMONS

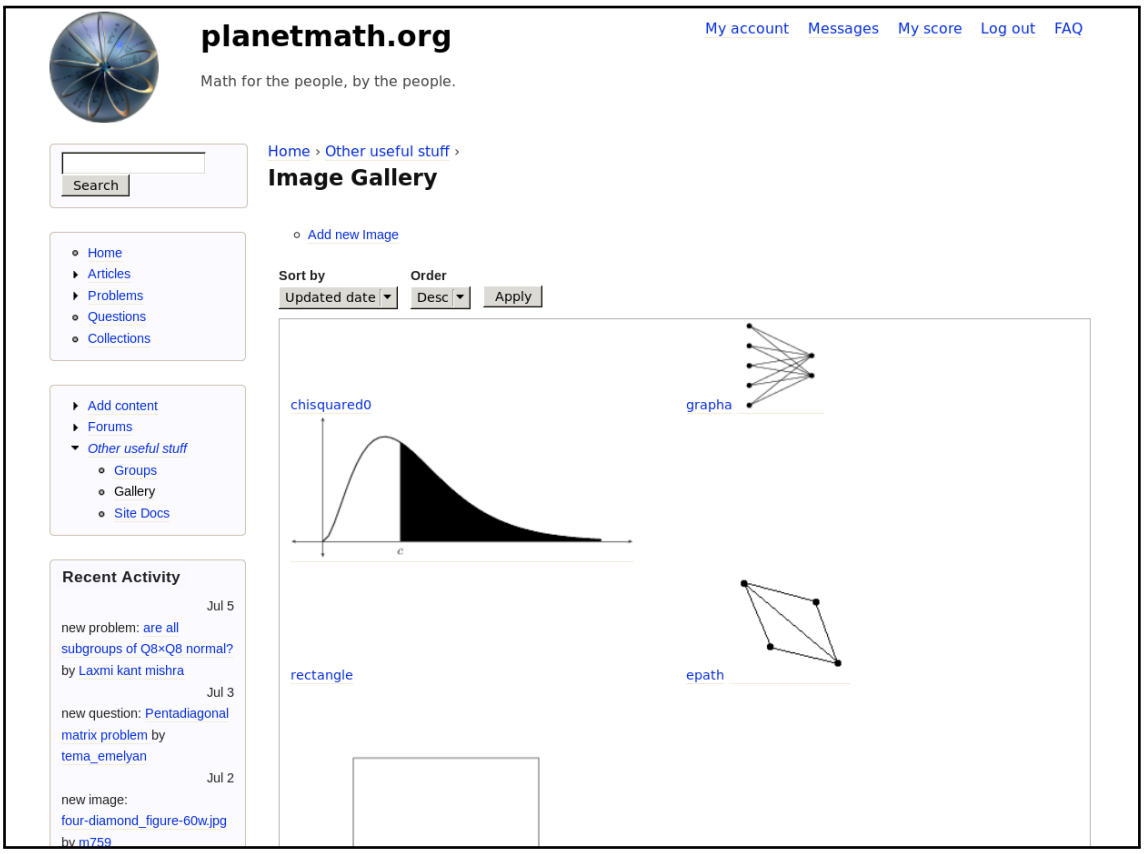


Figure 24: *A page from the new image gallery*

Figure 24 shows a page from the new image gallery. The look and feel is currently quite basic: in fact, the gallery is implemented as another Drupal view. Individual images can be included in articles with the LaTeX `\includegraphics` command. When including an image in an article, LaTeXML also deposits RDFa into the page to indicate the inclusion relation to downstream processes. In general, RDFa is useful when we need explicitly work with semantic details of user-contributed materials that would otherwise remain hidden in text. To illustrate one potential application: Since we know which articles use a given image, it would be possible to bring up images in search results by using the texts from the articles that include a given image as a source of index terms in the search engine. A content-focused next step would be to enhance images with instructions on how to generate the image: this is one place where a large set of examples would be very useful. Some recent thinkers on mathematics education have proposed to expand the typical acronym "STEM" (Science, Technology, Engineering, and Mathematics) to "STEAM" (Science, Technology, Engineering, Arts, and Mathematics) (Robelen, 2011). The current gallery is a modest foray into this space. It is also a prototype for other kinds of "shared" objects, for instance a centralized bibliography that would include citations from across the website in one database.

Enhance Groups are designed to help people organize according to their interests. The notion is that it will be easier to integrate user contributions to smaller domains, with the benefit of support from local experts. The gallery, or subsequent extensions, are analogous insofar as images that are shared between articles become a place to organize further work.

Obsolesce Groups partially obsolesce (or at least, make increasingly redundant) PlanetMath's front page, with its stream of posts from across the site. Relative to most specific goals, activity on a per-group or per-topic level is likely to be more coherent and in that sense, more meaningful.

Retrieve Groups offer the sense of a sub-system that is smaller than the whole, but larger than the individual. The aim with features like this is to retrieve a meaningful sense of relationship or even community.

Reverse If groups become strong in their own right, more than bringing people together, they could instead break up into little balkanized islands.

5. Support behavior tracking and semantic modeling

The reader will note that the "goal" and "strategy" data types proposed in Figure 1 were not implemented as specific data types. On the other hand, we've built features that give us the foundation for understanding goals and strategies as they are embodied in groups and collections. Thus, for instance, personal goals could be directly modeled as a collection of "things I'm working on." Paradata begins to convey a sense of progress. Collections can also be used to specify a sharable localized learning project, in the form of a reading list that helps build an understanding of a given topic. Teams and projects, along with an associated division of labor focused will be important if site contributors are going to successfully collaborate to build a rich paragogical learning environment. This point will be developed in more detail in Chapter 9.

At the level of domain modeling, one place to begin is with the Mathematics Subject Classification (MSC) is a taxonomic domain model for mathematics. Since the early days of PlanetMath, it has provided a way to browse the site's content "by subject." Reimplementing PlanetMath's MSC browser was an opportunity to develop and demo an integration between Drupal, LaTeXML, and the Virtuoso triple store (Lange et al., 2012), a first step toward exposing more of PlanetMath as Linked Open Data. This infrastructure is not yet integrated with the site's features for behavior tracking, although it would be desirable to do so. This applies to both collection-level paradata described above, and to site-wide activity tracking that uses the userpoints module.

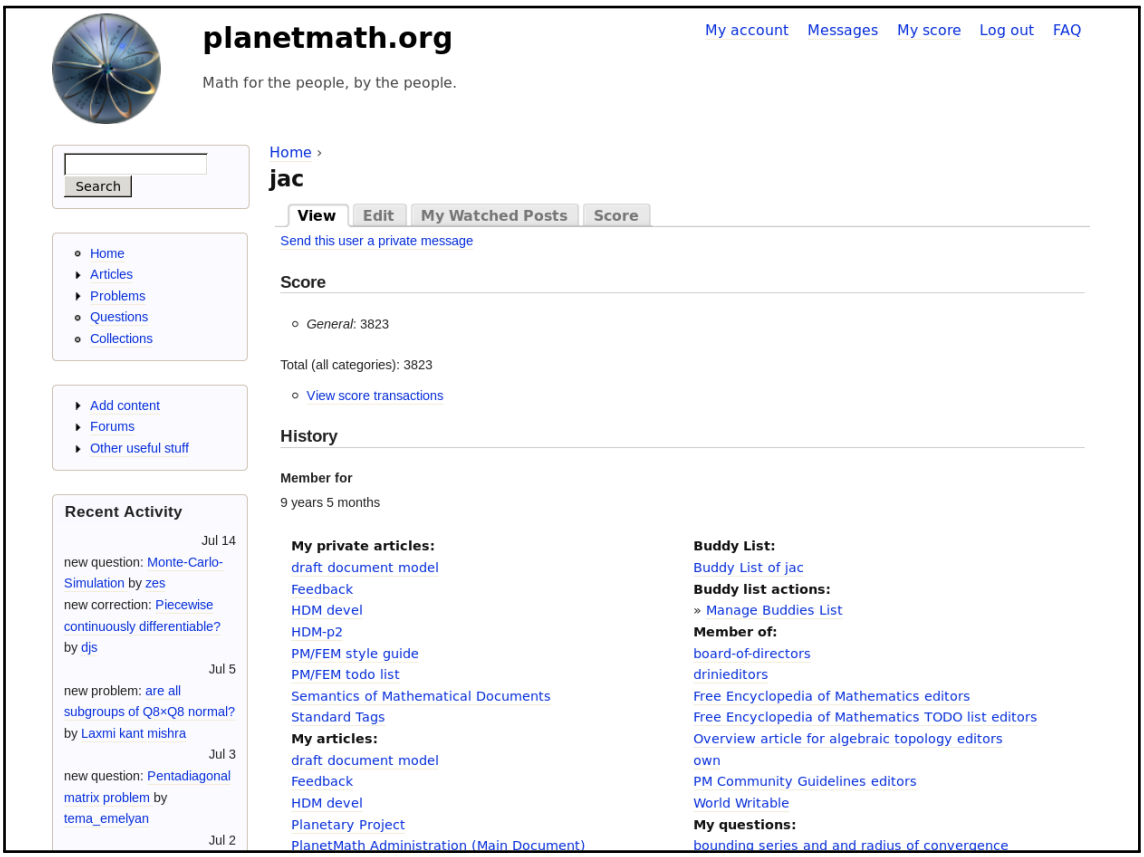


Figure 25: *User homepage: showing contributed content and the new buddy list feature*

The user homepage provides a simple listing of the user's contributed content (Figure 25). The user's private articles are only shown if the person viewing the user page is either the user that the page describes or a site admin. Basic biographical information appears at the bottom of the profile page, if the user chooses to supply it.

The user homepage includes links for managing the user's buddy list, as described above. It also includes links for managing watched pages and for sending private messages, which are supplied by the watcher and privatemsg modules, respectively.[*,†] The user's score is managed via the userpoints module; a summary of point-scoring activities is available via the "Score" tab on the user page.[‡] Editing the user page gives the user control over default settings that do not appear in this screenshot, including control over whether or not Planetary will use userpoints to track page reading.

Since some users have contributed hundreds of articles, a more ideal system would include tabs, tags, and other mechanisms for interactively organizing contributed content. This would help users keep track of and manage their contributed material, and help them control how it appears to others.

[*] HTTPS://DRUPAL.ORG/PROJECT/WATCHER
[†] HTTPS://DRUPAL.ORG/PROJECT/PRIVATEMSG
[‡] HTTPS://DRUPAL.ORG/PROJECT/USERPOINTS

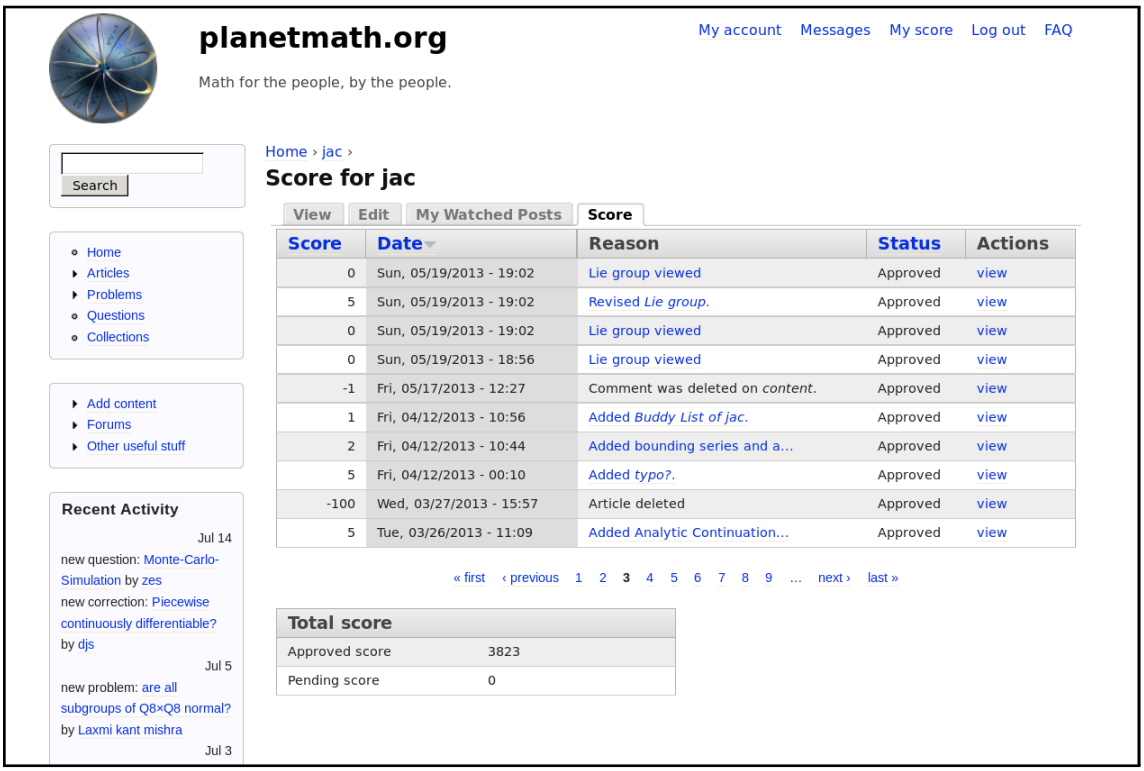


Figure 26: *Tracking individual actions with the* userpoints *module*

Figure 26 shows one of the pages from the "Score" tab, which contains a detailed breakdown of the user's contributions to the site. If the user has not opted out of page tracking, page reads will appear in the history with a contributed value of zero. Assigning and adding custom point values that correspond to particular interactions is quite easy with the userpoints module, which provides a function expressly for this purpose (`userpoints_userpointsapi`).

This record includes all of the substantial actions of the users, in some cases going back over a decade (although page reads were not logged in Noösphere). Additional interaction logging provides more data that is amenable to analysis using techniques from Chapter 6. However, to be useful within that framework, the data needs to be separated into vectors specifying the "treatment" and the "effect." This analysis could be made easier if activities were partitioned into meaningful "episodes," rather than simply collected into a large body of potentially unrelated work. Without user intervention, identifying the meaningful episodes and relevant causal factors remains a research topic, although it is one that has been investigated for some time; *cf.* (Faro & Giordano, 1997). One immediately useful idea at the level of user interface would be to display the history of interactions visually, using a module like timeline or views_timelinejs.[*,†]

[*] HTTPS://DRUPAL.ORG/PROJECT/TIMELINE
[†] HTTPS://DRUPAL.ORG/PROJECT/VIEWS_TIMELINEJS

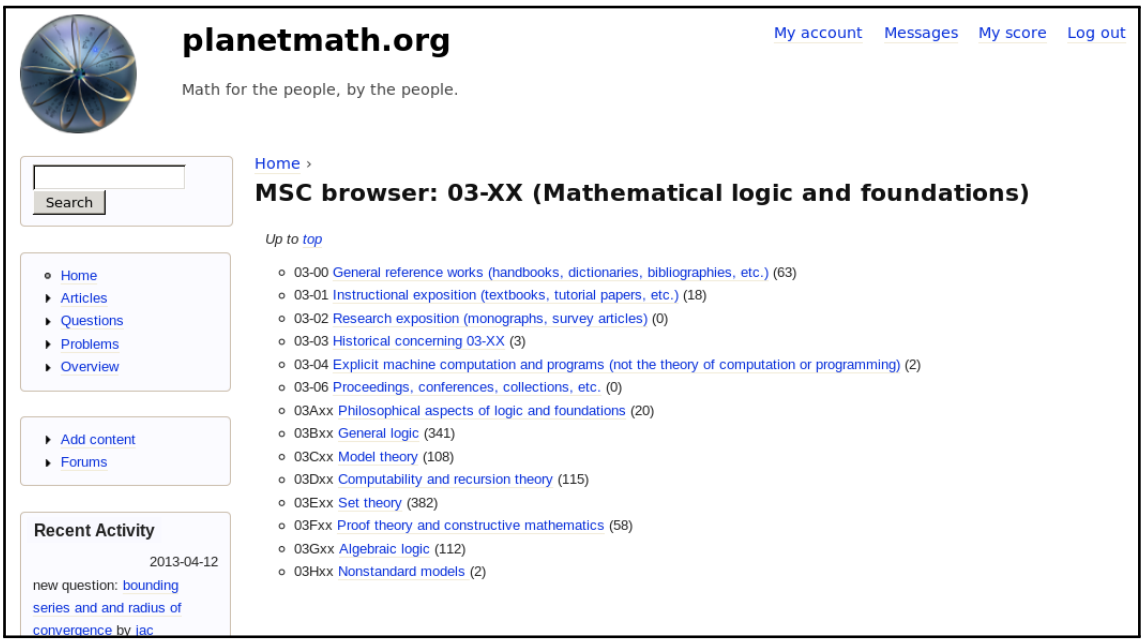


Figure 27: *Browsing articles via the Mathematics Subject Classification*

The primary source of top-down structure in PlanetMath/Noösphere was the Mathematics Subject Classification. This system has been replicated in Planetary, but with a new backend that will allow considerable expansion. Figure 27 shows the user browsing the encyclopedia by subject.

For the MSC implementation, we opted not to use Drupal's built-in support for taxonomies,* but instead used the new representation of the MSC as a Simple Knowledge Organization System (SKOS) ontology (Lange et al., 2012), served by an instance of the Virtuoso triple store that we connected to Drupal. Browsing the MSC generates pages and counts articles in various categories using SPARQL queries (Figure 27 shows the category 03-XX and the number of articles in each of its subcategories). The SKOS version of the MSC now includes MathML and multi-lingual category labels. Since the MSC is a core navigation feature, this is an important step towards eventual internationalization of the site.

Drupal serves pages as HTML+RDFa, where the RDFa we're most interested in is generated by LaTeXML from the LaTeX source code, which is further enhanced with links by NNexus. When pages are saved, we run a hook in the new pyrdfa module, which is a wrapper for a local instance of the pyRDFa distiller.† The distilled metadata is then sent to Virtuoso. This allows the system to keep track of which pages link to one another, which articles include which images, which articles are in a given category in the MSC, and to keep track of other provenance-style metadata like author and title.

*https://drupal.org/documentation/modules/taxonomy
†http://www.w3.org/2012/pyRdfa/

Using a triple store opens the way to clever ways to browse and recommend content, for instance, using nearby MSC categories to find related problems. We may also be able to use NNexus to assign MSC classes to otherwise unclassified texts, like forum posts and contributed problems, by clustering the MSC classes of the text's technical keywords. We could then index this content in subject-specific feeds, or attach related forum posts to similar articles or problems as a light-weight recommendation. One of the key issues here is disambiguation of terms with multiple technical definitions; *cf.* (Ratinov, Roth, Downey, & Anderson, 2011).

As mentioned above, currently quite a bit of relational metadata (and paradata) is maintained in Drupal and the underlying MySQL database rather than the triple store. This includes personal history data gathered with the userpoints module, and the hypertextual network of interlinked articles, problems, and solutions. This additional data will eventually be required for inference and recommendation facilities; the approach taken by Mulholland et al. (2013) is similar, and relevant code for moving data to and from the triple store could be reused.[*,†] Once we have a large enough collection of problems with induced MSC categories and depths, we should be able to recommend easier or harder related problems, taking both individual and aggregate paradata into account [§2.3, p. 31]. These models could build on literature related to adaptive testing (van der Linden & Glas, 2000) and the psychology of the "*growth mindset*" (Dweck, 2000).

Enhance More explicit object types together with more data on user interactions adds to our ability to use diagnostic tools, like the one described in Chapter 6. Additional user data around problem solving and peer support could provide the basis for a detailed study of learning-related effects.

Obsolesce Detailed record-keeping on PlanetMath could potentially obsolesce mechanisms like the course transcript or list of publications. As a history of work, the data we can gather resolves to a finer level of detail than either of these.

Retrieve As the system gathers more information about individual contributions, we gain a better picture of who our users are. A user's interaction history should not simply be thought of in terms of pre-determined "badges" or "achievements." It provides something more like the index to a laboratory notebook; *cf.* Bradley (2009).

Reverse If activity on PlanetMath becomes reified as a mechanism for producing value, we may see further effort expended towards mining this value, after the manner of the computations that drive the market for Bitcoins.

[*] https://github.com/ekilfeather/open-storyscope
[†] https://github.com/kmi/decipher

8.5 Preliminary system evaluation

A user-focused evaluation of the system is detailed in the next chapter; here the focus is on the technical aspects of the system in their own right.

When compared with Noösphere, Planetary is much easier to install, configure, and extend. Drupal's profile subsystem allows Planetary to be set up "out of the box" as a course management tool, or as the home to a larger PlanetMath-like learning community. Drupal's content management features, like the ones that we've used to build the groups, collections, and problem set workflows, help to set Planetary apart from other mathematics-capable tools. In addition to documentation generated by our team, many members of the worldwide Drupal developer community are happy to help new Drupal developers. The features we've developed cast PlanetMath's original mission – to be "*a central repository for mathematical knowledge on the web, with a pedagogical slant*" – in a new light.* Several legacy feature requests have been addressed in the process – and work on others is in progress.†

One could argue that the *combination* of features from other sites (*e.g.* Wikipedia plus StackExchange) could provide a system with the same functionality as PlanetMath. To some approximation, that is certainly true. This begs the question: what benefits do we get from putting various features under one roof, with a purpose-built system? The primary benefit is that integrative functions can be performed more easily. This is demonstrated well with the new collections feature. In the future, it would be attractive to increasingly integrate the platform with what others are doing elsewhere on the internet. For example, NNexus could be used to index mathematics blogs, and provide a topic-specific blogroll attached to each page in PlanetMath's encyclopedia (Ginev & Corneli, 2014): recall that the usual meaning of "planet" on the blogosphere is planet-as-aggregator.

I would cite two particular points of difficulty associated with using Drupal 7 as the basis of the new site:

1. *Drupal is known to have a steep learning curve.* This slowed our progress at the start, and sometimes things that should in theory be simple still ended up not working at all (thus, for example, dynamically changing menu items would work for a while, but then crash the site, so those features had to be turned off).

2. *We had to patch Drupal's core.* This is justifiably "taboo" amongst Drupal programmers, since it makes updating the site difficult, but we needed a new hook (`hook_post_save`) to supply our RDFa extractor with *rendered* HTML+RDFa; we also

*http://planetmath.org/history.html

†https://github.com/holtzermann17/planetmath-docs/issues/62

needed to modify Drupal's database interface to add support for the characters that are needed to display mathematical symbols.

Some of the technical issues will be resolved in Drupal 8 – but the system doesn't show signs of becoming simpler to learn. Luckily, the Planetary developer team has accrued significant wisdom, some of which can be passed along to future contributors. There are some "add-ons" for Drupal that are quite helpful for development work – for example, Drupal is fully scriptable via Drush,* and it can be interacted with using a Read-Eval-Print Loop (REPL), care of Boris.†

As a matter of building a usable system that can also be the basis for future work, the project has been a technical success. There are many places where new features, ranging from format shifting to research on recommenders for learning would be useful. Planetary is interesting partly because of what it makes possible – including the user study presented in the following chapter, which will help evaluate and better understand the paragogical model embodied in PlanetMath/Planetary. Future work on the system will progress subject to demand. As one small example: the activity data we store is similar to the format proposed in the ActivityStreams project, but use-cases for downstream consumption of data about activities on PlanetMath are not yet entirely clear, so we have not yet put effort into serving bona fide Activity Streams.‡ When the use case is there, the system is ready to adapt accordingly.

8.6 Summary

The Planetary system is an extensible open source platform for doing mathematics on the web. PlanetMath, which was previously oriented around authoring and editing a mathematical reference work, has been rebuilt using the Planetary platform. The new system hosts online problem solving interactions in a knowledge-rich, wiki-like environment. New features include custom collections, which can be used to build reading lists and interactive problem sets. Planetary records information about problem solving and other interactions, making it a useful tool for learning more about the way people learn mathematics.

While PlanetMath/Noösphere considered logical correctness, encyclopedic coverage, and expository quality to be matters of importance, mathematical problem solving did not receive explicit attention. The implementation work presented in this chapter opens new horizons for the PlanetMath community, with a focus on relevance and usefulness for a broader population of users. The development work has been informed by con-

*http://drush.ws/docs/shellscripts.html

†https://github.com/tobiassjosten/boris-loader

‡http://activitystrea.ms

tact with system users and stakeholders and by the theory of paragogy developed in earlier chapters. As a computer program, Planetary must follow strict rules; if anything, mathematics "*is much less formally complete and precise*" (Thurston, 1994). However, its expressive thickness derives not from the fact that all of the semicolons are in the right place and most if not all of the critical bugs have been eliminated – although this is certainly important – but rather from the fact that it can now be used, critiqued, and improved.

The discussion of screenshots framed using McLuhan's tetrad helps to add media-theoretic depth to the design dimensions and user desiderata discussed earlier in the chapter. The screenshots are, in a way, figurative depictions of typical mathematical contexts, but with a difference. In offline mathematics we are familiar with scratch paper, journals, and blackboards for building, sharing, and discussing mental models; gradebooks, transcripts, and CVs for keeping track of progress; problem sets, textbooks, and monographs for organizing learning materials; and libraries for browsing existing mathematical works. Planetary does not reproduce these technologies flawlessly, but the parallel online context has the advantage of being connected to a robust meta-level discourse and an ongoing process of technological change. This chapter has described the progress from design elements to working code deployed on a public website. The next chapter will explore the system's underlying dynamics in more detail, by examining the user experience.

9 User Evaluation

Qualitative findings from a user study after the public release of the new system demonstrate the feasibility of the paragogical approach, while suggesting a range of improvements to the software that will be needed for the project to successfully scale up – in particular, the further re-structuring of PlanetMath's activities into "sub-projects."

9.1 Research design

Planetary was deployed to PlanetMath.org on February 13, 2013. In April, I began recruiting participants for a 10 day study on PlanetMath. From emails sent to 15 all-time top-scoring and 30 other recently active PlanetMath users, I successfully recruited three persons. I also recruited three postgraduate students in mathematics at The Open University, contacted through their department.* The main question I focused on in the interviews was: would PlanetMath be relevant to the way you study and do mathematics? I also tried to elicit any ideas that might help to improve the site. Participants in the study obliged, with quite a bit of constructive critical feedback about the system. For example, one noted his resistance to creating collections, another talked about difficulties working with LaTeX, a third pointed out that PlanetMath would be more useful if it provided a feature for blog hosting, a fourth suggested new features for navigation. Figures 28 and 29 sum up the research design that helped to give order to this type of feedback, using a visual metaphor. The study aims to describe the relationship of system features to emergent patterns of use, here visualized as "paths in the grass."

*One person also joined in response to an announcement in the *Peeragogy in Action* Google+ community, but he dropped out of the study before completing an interview.

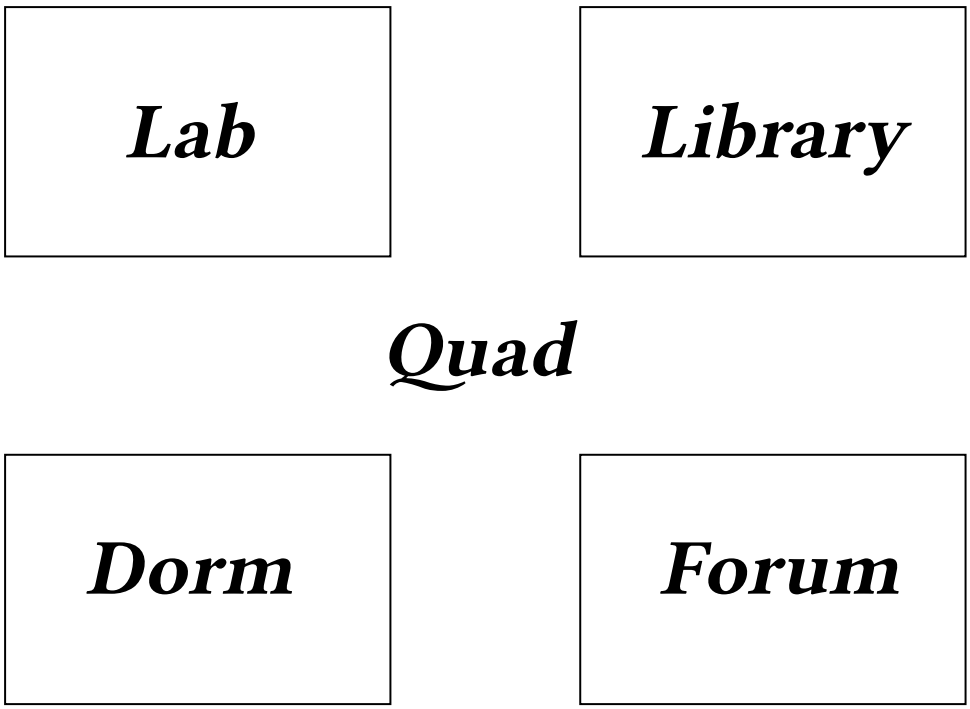


Figure 28: *Map of a virtual campus*

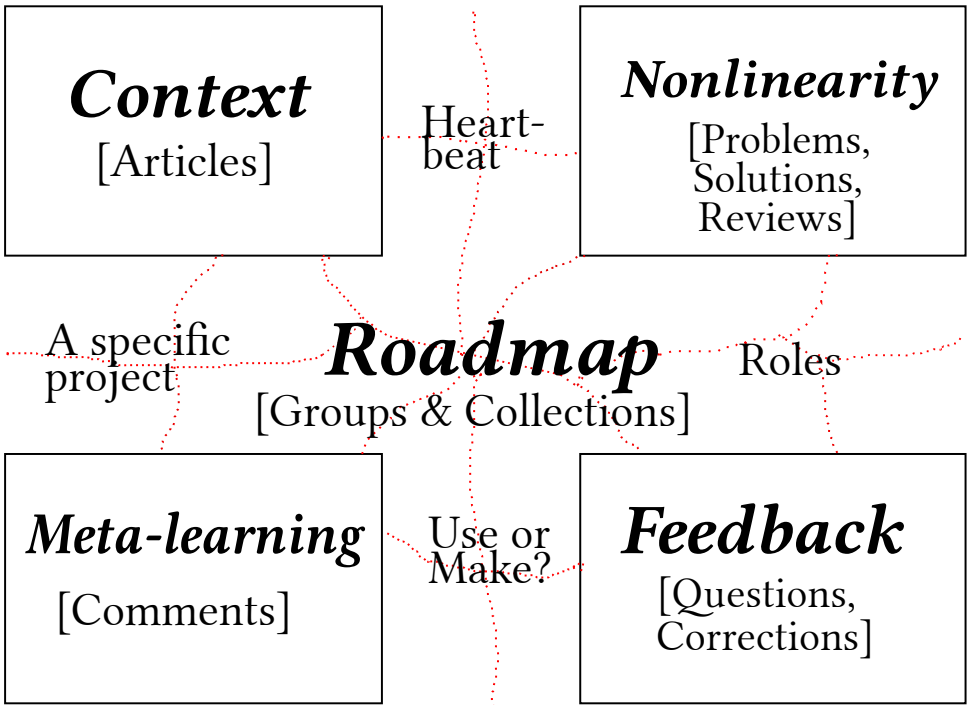


Figure 29: *Peeragogy patterns as loci for "paths in the grass"*

The Peeragogy project's catalog of patterns for peer learning (summarized in Table 13 [§5.2, p. 79]), which is used to seed the current study. If actual behavior were as depicted in Figure 29, we would expect to see people moving between "Articles" and "Problems" by way of "*A specific project*", or from "Question" to "Collection" by way of a division of labor among several "*Roles*." At a narrative level, this makes a certain amount of sense. In practice, the emergent structure of use may be more elaborate. More than a collection of hypotheses, Figure 29 represents a "meta-model" that can be used to identify the dynamic patterns of use that emerge within in a socio-technical system.

David Cavallo explained the idea of *emergent design* as a practice of "*probing for skills and knowledge resident in a community and using these as bridges to new content*" (Cavallo, 2000A). The aim here is to find not just new content, but new form – which necessarily involves transformation.

The study's phases are described in Table 19. In the first phase, participants (Table 20) attended a focus group where the study was described in detail, and where participants were asked to describe their mathematical backgrounds and to say what interested them about the study. I also walked them through a sequence of screenshots similar to those in Chapter 8. In the second phase, participants tried the new features of Planetary, on the public PlanetMath.org website. In the third phase, I conducted individual interviews. The participants were recruited in two cadres, so these three phases all ran twice (on May 2, 2013 and May 15, 2013). Participants were awarded a £32 Amazon.co.uk voucher upon completion of the study.

The analysis focuses on material gathered from the interviews; selected quotes appear in Appendix D. The initial analysis connects this feedback to the specific features of Planetary by way of the dimensions of design discussed in Chapter 8. Patterns offer another heuristic method to understand and organize new ideas coming from users. At the outset, several patterns (*Heartbeat*, *Roles*, *Use or make*, *A Specific Project*, and *Roadmap*) that came up particularly frequently in discussions in the Peeragogy project seemed likely to describe the user experience here. During interviews, I used the layout from Figure 29 as a template for visual notetaking.

A pattern-based analysis is rigorous in the sense that either the existing patterns match the topics under discussion, or new patterns should emerge. Design patterns are far from being a "universal" symbolic language in the Leibniz sense: nevertheless they can be used in the spirit of *mathesis universalis* as understood by Deleuze; cf. (Deleuze, 1968, pp. 229–230, 241). Design patterns can help build culture where before there were only constraints, symbols where before there was only a landscape; *cf.* [§1.4, p. 10]. In the course of analysis, the user evaluation of the site was encapsulated in three new design patterns that contribute to a new, emergent, design for the site, including concrete indications of "What's Next".

1	In an initial focus group, I will present the background of this study.	I will aim to discern what the *Roadmap* for individual participants looks like.
2	I explain the tasks that I would like people to try over a 10 day period, and explain that I will be available for discussion in the forum or privately by email.	I will aim to discern whether people use the four basic patterns (*Heartbeat*, *Roles*, *Use or make*, *A Specific Project*) as they orient themselves to the new software.
3	I will meet with each of the participants for an individual debriefing after the 10 days has passed.	I will aim to discern whether people use any other more refined patterns as they used the tools.

Table 19: *Phases of the structured component of the user evaluation*

AA	UK resident, male, chemistry background, currently working in media; hoping to enroll in the Open University's mathematics masters degree (contacted through PlanetMath)
AB	UK resident, female, current postgraduate student in mathematics (contacted through OU mathememathics department)
AC	German resident, male, mathematics PhD, staff scientist and programmer, long-time PlanetMath user (contacted through PlanetMath)
RSP	US resident, male, physics PhD, long-time PlanetMath user and active volunteer (Raymond S. Puzio, requested non-anonymity)
BA	UK resident, male, engineering background, enrolled in the Open University's mathematics masters program (contacted through OU mathematics department)
BB	UK resident, female, engineering and software background, enrolled in the Open University's mathematics masters program (contacted through OU mathematics department)
BC	UK resident, male, teaching and media background (contacted through the "Peeragogy in Action" Google+ community; BC dropped out of the study partway through)

Table 20: *Study participants*

9.2 Thematic analysis

Individual interviews offered the most information and are examined in detail in this section. Detailed quotes from the interviews are presented in Appendix D and are referenced here in square brackets.* The analysis also draws on observation of participant interaction with the tool, as well as forum posts and emails exchanged with participants during the study. The most useful way to begin to organize the analysis is under the headings coming from the initial dimensions of the design (Chapter 8), and to think in terms of the concrete system features that instantiated this design (Figure 14 [§8.3, p. 114]). The key ideas are collected into emergent themes.

1. A special-purpose tool customized to suit the challenges of learning mathematics

"Necessary but not sufficient" In our interview, **AA** – who came to PlanetMath with mostly undergraduate-level questions – said that the site is only as good as the *people* who use it. **AB** – who was thinking about research-level problems – suggested that it is only as useful as the *content* that it provides. Working software is clearly another "necessary but not sufficient" criterion. A model devised by **RSP**, connected with earlier discussions among core volunteers in the PlanetMath project, came up again in our interview [RSP1]. A schematic rendition of this model is presented in Figure 30. *Content* and *Community* features are familiar from other social websites. A *Catalog* is similar to what one finds in libraries: a resource that provides pointers to other available resources. **RSP** is particularly interested in using existing library catalogs to find additional content that can be used on PlanetMath [RSP2]. Inasmuch as the catalog helps to find content, it represents an essential feature. **BB** repeatedly emphasized the importance of functioning search – otherwise we will have "*masses of very good math, masses of very good theorems and results and definitions and so on*" – that are effectively useless because you can't find the one you need [BB1]. Thus, a tool that supports learning mathematics should at least support these three core features. Two additional components, *Code* and *Organization*, provide the basis for the existence and evolution of such a tool, but most users will not interact with these components directly [RSP1].

Challenges with writing mathematics At the moment, the most prominently visible mathematics-specific aspects of the system have to do with *content* – and particularly with entering and displaying mathematical *symbols* in textual form – although mathematical *figures* were also discussed. As it stands, even mathematical symbols presented

*Recordings from **AA** and **AB** are missing due to a problem with the recording software; accordingly, data from these interviews is embedded in diagrammatic form in my handwritten notes. **BC** dropped out of the study before completing an interview.

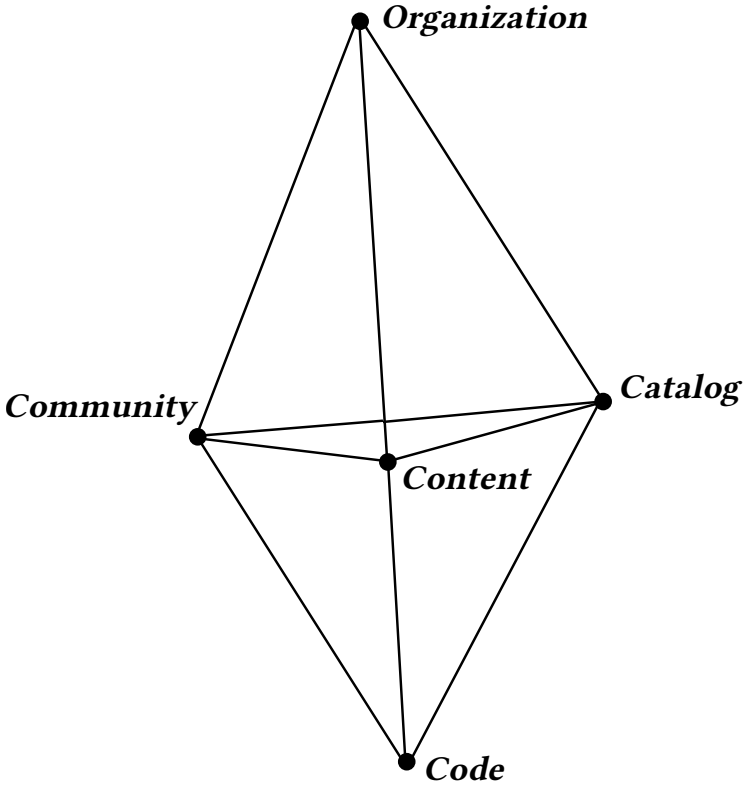


Figure 30: *Raymond Puzio's "3C" model, with elaborations from our conversations*

various difficulties for some of the users. **AA**, who was new to LaTeX, emailed me several times during the study asking for typesetting help. **BB** and **BA** were familiar with the "What You See Is What You Get" (WYSIWYG) LaTeX frontend LyX,* having used it to complete their course assignments. They managed with PlanetMath's current web editor, but **BA** indicated that he missed the familiar WYSIWYG workflow. With the current web editor, he felt concerned that the browser might time out, and that he would lose any text that hadn't been saved [BA2].

Currently, LaTeXML "*fatal errors*"† (where no resulting HTML can be generated) aren't reported to the user at all and result in a blank screen, causing a strong sense of uncertainty and doubt when things go wrong – even for an experienced LaTeX user like **AC** [AC1]. On the other hand, **AC** emphasized that having LaTeX itself is quite useful [AC2]. Autosave and auto-preview would not be terribly hard to add to the current PlanetMath editor, particularly since both the editor and the LaTeXML daemon have WebSocket support. Simple point-and-click entry of mathematical formulas would also be an advantage for new users. Novice users might also prefer to use handwritten mathematics (Lo, Edwards, Bokhove, & Davis, 2013). **BA** indicated that scans of handwritten papers can be submitted in Open University courses, which lowers the barrier to participation.

*http://www.lyx.org/

†http://dlmf.nist.gov/LaTeXML/manual/errorcodes/

"Nice to have" In the interview with **AB**, we came up with a way in which an improved gallery could be useful for search. She was looking for a paper in graph theory – but wasn't sure what search terms to use, although she knew what the graph she had in mind looked like. Our idea was that if PlanetMath's gallery had more semantic structure, allowing her to find the figure, it could also be used as a way to find the relevant definitions. Currently, however, the gallery needs many basic improvements, and image search does not work at all. **BA** found the workflow of uploading to the gallery difficult and suggested that images wouldn't often be reused and that the global gallery was of limited importance. He suggested to switch to "article-local" interaction with images (as in Noösphere). It should be possible to support intuitive local uploads that integrate with the global gallery. As time goes by, we may see user interest in additional features for remote collaboration, including various mathematics-specific features, like a interactive computer algebra system [RSP3].

2. An active and interactive process centered on problem solving

Progressive problem solving Several subjects brought up interesting points and questions about the meaning of "problems" and "solutions." This helped to show that the entity relation diagram in Table 10 [§5.2, p. 74] could be made more complete by adding conjectures and partial solutions [AC6]. It seems clear that interactions centered on research problems and textbook problems would be rather different. **BB** outlined an ethical stance on questions about homework problems [BB2] that is very different from **AC**'s orientation towards sharing both questions and partial answers to research problems.

BB's understanding of the right thing to do in the case of *questions about homework problems* was consonant with the design decision to have "answers" be links to encyclopedia articles [BB2], [BB4]. Linking to encyclopedia articles is effectively a hint, indicating relevant and ostenibly common knowledge, which should put the questioner on the right track, but which may still leave them something to work out. However, when it comes to *her own solutions to problems*, like **AC**, **BB** was comfortable with uploading and discussing conjectures and partial solutions as work-in-progress [BB3].

It was also clear that unless users are comfortable sharing incomplete answers, partial, or failed solutions, then this sort of knowledge will be underrepresented – which may represent a missed opportunity for learning (Tsovaltzi et al., 2010). Nevertheless, there is a barrier to overcome. **AB** said she found it nice to be able to help – but she was concerned that if she admitted that she couldn't figure out the answer to a given question, or if she posted an incomplete answer, people would get upset. She reported spending an hour attempting to work out a tricky integration by parts problem that another user had uploaded, before giving up – no trace of this work showed up on the site. Regard-

ing another question having to do with differential equations and interest rates, she said she would have liked to help, but "*I haven't looked at that in years.*" Certain comments from **AB** and **BB** were aligned with an idea that I had previously discussed with the Planetary development team, namely the creation of an Etherpad-like website for mathematics [BB3]. This would be a stand-alone system built using an improved version of the PlanetMath editor for creating ephemeral (potentially anonymous or throw-away) mathematical artifacts, the electronic equivalent to a chalkboard. However, if this collection was (optionally) taggable and searchable, the material could be re-used, and could, furthermore, provide a real-time guide to topics of interest around the site; *cf.* (Corneli, 2010), (Mary, 2013).

Personal history, constructivism Planetary currently presents an overview of the user's "History" on the user homepage. It tracks more detailed interactions using the userpoints module, but **AC** found the presentation of this information confusing [AC11]. **BB** pointed out that history is not only important for the user home page, but could be usefully integrated into the browsing experience, using appropriate breadcrumb trails rooted at important topics [BB7]. Meaning making is associated with "constructive" features, and collections represent a light-weight tool for constructive meaning making. Subjects generally agreed that collections provide a useful way to organize material, and also that there were various ways they could be improved, for instance: by adding the ability to "expand all"; by being able to display sub-collections in a nested format; by being able to include links to outside resources; or by adding narrative "glue" between components. For **BB**, who was thinking primarily of reference-related user interactions, PlanetMath's existing narrative *topic articles* that summarize important mathematical topics present a compelling alternative to collections [BB8]. **BA** felt that collections should be managed by someone with a designated "editor" role [BA3].

3. Motivate users to contribute problems and discuss solutions

Regulating learning in a social/mediated context Although she described herself as "*not very self-motivated*," **BB** seems to be highly motivated in the right context – and she is very interested in understanding how she learns best [BB5]. After our interview, she sent me a description of her own learning patterns [BB6], which seemed to closely match the "*forethought*", "*performance*", and "*self-reflection*" cycle from (Zimmerman, 2002; Zimmerman & Campillo, 2003). As Zimmerman and Campillo (2003) indicate, there are differences between problem solving in "*formal*" and "*informal*" contexts. Although mathematics is a relatively formal domain, PlanetMath can also help people solve a range of "*ill-structured*" informal problems. For example, **AA** was particularly enthusiastic about PlanetMath's potential as a place to network with people with similar interests, but pointed out that there needs to be a "*sense of urgency*" about answering

questions that are uploaded to the site. I mentioned that the StackExchange system seems to have done a good job in that respect. **RSP** also expressed interest in networking [RSP4], but pointed out that simply displaying a log of user activities wouldn't really be sufficient for finding collaborators, since even very prolific contributors "*might not be in the mood to talk about it*" [RSP5]. He suggested that a map of user activity would be more useful if it included more detailed social networking information, beyond just mathematical interests or a state of "co-presence" online [RSP5], [RSP7].

Comparison with roles in other contexts People are motivated to use systems that help them achieve their aims. People are relatively *unlikely* to report software bugs if they don't have a sense that the bug reports will be responded to. I encouraged **AC** to point out any software problems he encountered with PlanetMath/Planetary, even though I had to admit that the Planetary development team is not "*available 24/7*," [AC7]. On the software level, if we make it easy to report bugs, if we respond to bug reports, and if we make it easy for others to get involved with the software, then there's a good chance that the software will eventually become satisfying for site users. "Bug reports" and "feature requests" present a useful model, not just for software, but for mathematical content – indeed, this is precisely what *corrections* and *requests* were supposed to accomplish in Noösphere. However it may be time to introduce a new abstraction layer, that helps people discuss and engender more broad-sweeping change (*i.e.*, above the level of individual articles, which is what corrections target). An "open" production model comes with the ability to redesign the platform and content as a work-in-progress. This is quite different from the model used in OU courses, where, for instance, the forum moderator tries to "*dispel any talk*" about the Tutor Marked Assignments [BA6].

4. Support mathematical meaning-making

Readily available feedback Study participants pointed out in different ways that if there are to be interactions with other people, then people have to show up. **AC**, who had first joined the site around the time of PlanetMath's peak in activity in the mid-2000s (Figure 2 [§2.1, p. 25]) noticed that things were fairly quiet around the site these days [AC3]. He pointed out some questions that the PlanetMath ownership model poses regarding interaction with content once article owners have left the site [AC4]. Relating back to **AA**'s point about a "*sense of urgency*," **AC** noted that it is very important to be able to "watch" a problem, so as to get notifications when its status changes [AC5]. In OU courses, the tutor is available for live help on an individual basis: as **BA** indicated, "*The student can contact them by email, phone – and, um, whatever.*" We can contrast this with the experience in PlanetMath, where, for instance, an active volunteer like **RSP** expresses a somewhat distant hope for time to help newcomers [RSP10].

Concreteness as a criterion of quality Concreteness applies at both the system and content levels: people need to know what they can do, and what it means. Some of the study participants found the proliferation of options and types of data on the home page confusing. It seems possible that something like a "start menu" would help people see what they *can do* on PlanetMath – and if someone wants to do something that isn't possible yet, they should be instructed to use the action, *send a message to the developers requesting a new feature.* **BA** described the OU courses he had taken, and told me that he had quit one of them part way through [BA7]. Even though "*some people*" can find their way around functional analysis, his course was presented in a way that made the topic seem "*so abstract*" that he was not able to follow it. **BA**'s experience points to the importance of being able to visualize the topic that is being studied, and understand how it can be applied. Adding concreteness is one "service" that a knowledgeable teacher or mentor can provide – if one is available.

5. Support behavior tracking and semantic modeling

Personalization and localization **AC** was particularly enthusiastic about the ways in which PlanetMath could, at least in theory, be personalized to the user's interests. In his view, PlanetMath shows more potential in this regard than sites on the StackExchange network [AC8]. This perception is striking particularly because there is still considerable room for improvement in implementing personalization features. Some of these features would be within easy reach, for instance, after the model of dashboard module used on Drupal.org.* **AC**'s perception of personalizability may have to do with PlanetMath's *openness* [AC9] and its subject-specific focus. He also had some specific technical suggestions. One of these was to incorporate blogging tools in PlanetMath user pages [AC10]. **RSP** suggested a similar point, namely, to incorporate familiar social networking tools into user homepages, using a system like Friendica† [RSP6].

One of the ways to localize around specific interests is to "segment" the site, per requests from legacy users, see Figure 13, [§8.2, p. 110]. **BA** suggested that people working at different levels may not need to interact much [BA5]. **BB** made a similar remark that indicated that segmentation would be most useful when combined with a localized, faceted search functionality [BB9]. Connecting this to the theme of personalization, **RSP** pointed out that all of PlanetMath's important features always "localize" to any given user at any given point in time [RSP8], although they may fail to do this gracefully. He noted that individual projects had not been particularly well modeled or well supported in PlanetMath/Noösphere [RSP9].

*HTTPS://WWW.DRUPAL.ORG/PROJECT/DASHBOARD
†HTTP://FRIENDICA.COM/

9.3 Pattern analysis

As a secondary level of analysis, the interview data is further analysed in relationship to the Peeragogy pattern catalog (Table 13 [§5.2, p. 79]). The notion of pattern-finding as a process related to, but distinct from abstraction, is described by Richard Gabriel, who emphasizes that the "*patterns and the social process for applying them are designed to produce organic order through piecemeal growth*" (Gabriel, 1996, p. 31). This criterion is here applied to the growth of the pattern language itself. This section draws on the thematic discussion from the previous section, and aligns user feedback with the existing pattern language, in the process of describing three new patterns. A discursive presentation is presented first, followed by a summary using template from Table 11 [§5.2, p. 76] in Tables 21–23, which appear at the end of this section.

Frontend and Backend

Although mathematics is a relatively formal domain, many of the motivations for using PlanetMath map onto Zimmerman and Campillo's (2003) dimensions of "*informal*" problem solving. Informal problems are "*are personally defined*" and possess "*open-ended boundary conditions*": they are situated within an open world. "Formal" motivations can typically be addressed by a generalized look-up approach: for instance, by reading the manual, or running the algorithm. An acquaintance with the formal features of mathematical problem solving is usually seen as a prerequisite for engaging constructively with the more informal activities of mathematics research, which involves scratch work, serendipity, and various human factors. This dichotomy suggest a new and important pattern, linked to the 3C+2 model developed in conversations with **RSP** (Figure 30). The pattern could be called *Frontend and Backend*. A similar distinction was proposed by Reuben Hersh (1991), after Goffman (1959). Hersh compares the "*front*" of mathematics to the seating area of a restaurant, and the "*back*" with the kitchen. In this metaphor, the suggestion is that the front is misleadingly neat and the back realistically messy. This corresponds with Thurston's perspective on written mathematics [§4.2, p. 49].

However, rather than focusing on degrees of formality, the *Frontend and Backend* pattern is perhaps best understood in relation to the *Newcomer* pattern. Typically one will not expect the user of a system to know how to, or be motivated to, work with backend features until they have mastered at least some of the frontend features. For example, you have to eat before you can cook, and it would be rare to find an auto mechanic who did not know how to drive. David Cavallo (2000B) wrote about an "*engine culture*" in rural Thailand in which structurally open systems brought aspects of what residents of more modernized regions think of as "backend" features into the "frontend". Thai farmers routinely move one engine between different pieces of farm machinery, for

example. In mathematics, textbook problems are stated in terms of a certain vocabulary, but for all but the simplest problems, knowing the definitions of the terms is not enough to solve the problem. One needs to understand how the ideas fit together. The *Frontend and Backend* pattern clearly lends itself to standard service provision, as, for example, in Open University courses. It can also be part of paragogical activity, as in **RSP**'s proposal to focus energy on supporting individual users, by helping them develop a high-quality sub-site on their topic of interest [RSP8], [RSP10]. This would simultaneously inform the development of various replicable backend features, and help raise the profile of the site as a whole. The pattern is in this way associated with *A Specific Project* and with the *Roles* pattern. *Frontend and Backend* also connects with the *instrumental* and *interactional* modes of engagement discussed in Chapter 7 [§7.2, pp. 94-95]. If you only want to know the answer, or if you wish to avail yourself of a service that can be rendered in a transactional manner, then you can usually stay at the level of the frontend. If you want to understand what's really going on – or if the service can only be rendered interactionally – then you may have to take a look at what's happening in the backend.

Spanning Set

This pattern connects with **BB**'s thoughts about using topic articles as a map to the content. You may be able to get what you need without digging – but if you do need to dig, it would be very good to get some indication about which direction to dig in. **BB**'s proposal is similar to an idea described in a semantic web context by Katerina Tzompanaki and Martin Doerr (Tzompanaki & Doerr, 2012A, 2012B).

> *In order to overcome the problem of effective searching* [...] *we propose a querying system for semantic networks based on a few fundamental categories (FCs) and (binary) relationships (FRs). These categories are "base classes" covering the domain, and the relationships are deductions from complex path expressions of all sorts of deep relationships and documentation alternatives in a much richer and more specialized semantic network*[.] (Tzompanaki & Doerr, 2012A)

BB's proposal and the interpretation above point to another pattern, related to but distinct from *Frontend and Backend* – the *Spanning Set*. This may be made up of media objects, or of people. In a standard course model, there is one central node, the teacher, who is largely responsible for all course communication. In OU mathematics courses, this model is scaled up; as **BA** indicated,

> [E]*veryone's allocated a course tutor, who might take on just a half-dozen students – so, they're not the overall person in charge of the course, by any means.* [BA6]

Given sufficient demand, the model could be extended to courses or learning platforms

that are arbitrarily large. For instance, this can be done via a master/apprentice system, in which every apprentice is supervised by a certified master. In the typical online Q&A context, these roles are made distributed, and are better modeled by power laws than by formal gradations.

There are intermediate modes. Stephen Downes is correct that a *star graph* S_k (with k students and 1 teacher) does not work well for learning purposes as k grows.* This is a problem of *Carrying Capacity*. However, a (j, k)*-biregular graph*, in which every one of j teachers has k students does not have the same problems if j grows along with k. A given discussion group of 100 persons that is divided according to the so-called 90/9/1 rule,† would have 90 lurkers, 9 contributors, and 1 content creator. This is what one might observe, for example, in a classroom with a lecture format. The system would presumably shift if it was broken up into smaller groups. For example, suppose each of the 9 contributors leads a small discussion group of 10 persons. The worst case scenario for at this scale is a "9/0/1 rule" but it seems reasonable to expect "8/1/1" or better.

David Cavallo wrote that in his research with Logo, the goal "*is not only* [working] *for the benefit of others*" but also "*to help the developers themselves formalize and make robust the knowledge required to accomplish the project*" (Cavallo, 2000B, p. 127) This principle can be applied at every level in paragogical projects. It may be embodied by the members (or elements) of a Spanning Set. For instance, it would be worthwhile to make the PlanetMath encyclopedia into an effective Spanning Set for a wide range of mathematics questions. In some cases, a Spanning Set already exists in a concealed form. For instance, the PlanetMath project has an existing *Heartbeat*, in the form of quarterly board meetings and weekly informal discussions. This functions as a Spanning Set for staying up to date in the project's activities – for those who attend. It should be possible to scale up the informal discussions somewhat simply by advertising them more widely; as **RSP** remarked:

> [T]*hings like restarting the community discussions, for example our having our Thursday things once a month, with more people showing up. None of these really require us to build any new software or do anything too time consuming.* [RSP12]

Similarly, "Previews" of features-in-development would help illustrate existing priorities to people who may want to know what's coming next, and how they can get involved.‡ These examples show how in many cases the idea of a Spanning Set can refine the pattern for *Creating a Guide*. Making it clear what people *can* do will help keep them from having to go too far out of their way when it comes time to decide what *to* do.

*http://www.slideshare.net/Downes/the-connectivist-learning-environment (Slide 6)
†http://www.wikipatterns.com/display/wikipatterns/90-9-1+Theory
‡https://github.com/holtzermann17/planetmath-docs/issues?labels=PREVIEW

Minimum Viable Project

The Minimum Viable Product approach to software development is about putting something out there to see if the customer bites (Ries, 2011). Another approach, related to the pattern just discussed, is to make it clear what people can do with what's there and see if they engage. We might call this the *Minimum Viable Project*, an adjunct to the *Roadmap* pattern, and a new interpretation of the earlier pattern *A Specific Project*.

What makes a project "viable"? Beer (1981) emphasizes the feature of recursivity: viable systems are typically made up of smaller viable systems. This again suggests that one way to strengthen the PlanetMath project as a whole would be to focus on support for individual projects. The front page of the website could be redesigned so that the top-level view of the site is *project focused*. Instead of collecting all of the posts from across the site – or even all of the threads from across the site – the front page could collect succinct summary information on recently active projects, and list the number of active posts in each, after the model of StackExchange questions or Slashdot stories. For instance, each Mathematics Subject Classification could be designated as a "sub-project" and there could be many other cross-cutting or smaller-scale projects. This points to an important syntactic modification to Table 10 [§5.2, p. 74] – introducing the *project* as a container for other productions. Rather than having all of the chains show up in one container, each project would present a distinct context for constructions. Using the grammatical metaphor, relationships between projects would be similar to anaphora as a relationship between sentences.

A viable system is always "*organized with respect to what it is not*" (Deacon, 2014). In other words, each project will have a *Frontend and Backend*. This suggests that projects model their outcomes to some degree of fidelity, and that they are made viable by features that connect to the motivations and ambitions of potential participants. In addition to the basic feature of recursivity, Beer's (1981) dimensions of viable systems (which can be applied to study any project – or pattern) are:

1. **The elements that produce the system.**
2. **Information channels for communication between the elements.**
3. **Internal control/regulation systems.**
4. **Sensors and regulators along the outer boundary.**
5. **Policy decisions governing the system as a whole.**

Frontend and Backend: A separation of concerns

Definition: The part of the system users see is often connected to a "backend" that they don't interact with as much. Working with the frontend is more formal and rulebound, while working with the backend is relatively informal.

Problem: The idea of Frontend and Backend is related to the *Newcomer* pattern: typically one will not expect the user of a system to know how to, or to be motivated to, work with any of the backend features of a system until they have mastered at many of the frontend features. "Users" tend to expect a level of service provision – and *new* users often require some hand-holding.

Solution: The pattern of Frontend and Backend lends itself to standard service provision and transactional models of exchange. However, it can also be part of commons based peer production, if people are willing to put in the work. Sophisticated oldtimers can focus energy on supporting individual newcomers by helping them develop a high-quality suite of content and tools focused on their topic of interest. Reflection on this process can inform the development of the system backend. In addition, the new content can help to raise the profile of the site as a whole. This pattern is associated with focusing on *A Specific Project* (following the interests of the newcomers) and with the *Roles* pattern, since it requires a committed and knowledgeable mentor who learn from working with newcomers.

Challenges: Simultaneously mentoring newcomers and working on system features constitutes a major commitment. If this work can be spread out among several volunteers – or possibly paid staff – this could have some advantages. Depending on the nature of the process, providing a single point of contact for the user may still be the most straightforward.

Example: David Cavallo wrote about an *engine culture* in rural Thailand, in which structurally open systems made tinkering with the "backend" features of internal combustion engines a part of daily life. Cavallo observed that people who were familiar with tinkering with engines tended to be able to learn how to tinker with software, suggesting that there are some common underlying informal reasoning skills.

What's Next: At PlanetMath, we have an *open engine*, but not yet an *open engine culture*. Along with continued work to serve and understand specific users by focusing on specific and concrete use cases, we will want to build pathways for meaningful user involvement in work with the software system.

Table 21: *The* Frontend and Backend *pattern*

Spanning Set: Follow the paths in the grass

Definition: With a well-constructed information access system, you may be able to get what you need without digging. If you do need to dig, it is very good to get some indication about which direction to dig in. At the level of content, this may be achieved by using high-level "topic articles" as a narrative map to the content. In general, the Spanning Set may include people as well as less dynamic media objects. A spanning set is comprised of a set of fundamental actions (e.g. asking a question) and fundamental relationships between resources.

Problem: People need to know what can be done with a given resource, and this isn't always obvious. Relying on a single knowledgeable guru figure isn't always possible.

Solution: A system's features, categories, and relations can be comprised of many different kinds of components, and it needs to be organized in appropriate ways. For example, a "start menu" or pop-up window showing keyboard shortcuts can show what can be done with a given tool; a schedule of office hours can show people when and where they can find help; and topic-level narrative guides to content can show people where to read more.

Examples: *Creating a Guide* and *Heartbeat* both produce spanning sets. In a standard course model, there is one central node, the teacher, who is responsible for all teaching and course communication and answering student questions. In large courses, this model is sometimes scaled up through tutors and TAs. In a large MOOC, a "spanning set" of peer tutors could help give everyone personal attention. In Q&A sites, these roles are disaggregated and distributed.

Challenges: From the perspective of *Frontend and Backend*, principles and features are visible as part of a system's "frontend" – but the spanning set of relevant behaviors tends to be emergent. If any individual tries to span too much, they will get spread too thin; see *Carrying Capacity*.

What's Next: As a project with an encyclopedic component, PlanetMath can be used to span and organize a significantly larger body of existing material. The high-level idea is that of a "cross-index" to the mathematics literature. Initial prototypes should be built for smaller domains (for example, Calculus or Mathematical Biology).

Table 22: *The* Spanning Set *pattern*

Minimum Viable Project: Hard work for no pay? Where do I sign up?

Definition: The Minimum Viable Product approach to software development is about putting something out there to see if the customer bites. Another approach, building on the notion of a *Spanning Set*, is to make it clear to *Newcomers* what they can do with what's there, and see if they engage. Whereas a *Roadmap* lays out a plan for *A Specific Project* that could go nowhere, a Minimum Viable Project is one that people will actually engage with.

Problem: Minimum Viable Projects may have a some features in common, and these patterns could be studied using the features of viable systems in general. However, the proof is in the pudding, so we especially need a methodology for trying things out.

Solution: The proposal is to take a project-oriented view on everything. Understand actions and artifacts as being embedded within projects, modeling projects in terms of user experience and system features (per *Frontend and Backend*). Where possible, project updates can be modeled with a language of fundamental actions (per *Spanning Set*). Projects themselves model their outcomes to some degree of fidelity, and they are most likely to be viable if they have features that connect to the motivations and ambitions of potential participants. The practical side of this proposed solution will be to build systems that can express all of these aspects of projects, and study what works in practice.

Challenges: There are many different kinds of projects, operating at different scales, and with different kinds of actors involved. Each project will have its own *Frontend and Backend*, which will define interfaces to other projects, and this could get complicated.

What's Next: The front page of the website could be redesigned so that the top-level view of the site is project focused. That is, instead of collecting all of the posts from across the site – or even all of the threads from across the site – the front page would collect succinct summary information on recently active projects.

Table 23: *The* Minimum Viable Project *pattern*

9.4 Translating to a pattern-based design

The design dimensions that guided implementation in Chapter 8 can be reassessed in light of this study. Further work on the project can be translated into to a pattern-based design, centered on the three patterns summarized in Tables 21, 22, and 23, where connections to other patterns from Rheingold et al. (2014) are indicated.

Rather than describing the implementation project in terms of (1) *a special-purpose tool customized to suit the challenges of learning mathematics* that would support (2) *an active and interactive process centered on problem solving*, these ideas could instead be described using the idea of a *Spanning Set*. The tool is special purpose just insofar as it makes the actions that are relevant to learning mathematics sufficiently clear. For instance, the notion of a *Spanning Set* could be deployed to progressively clarify the complex notion of problem solving by focusing on the fundamental categories and relationships that make the search for solutions effective. At the heuristic level, a pattern catalog could be used for this purpose, and applied to reassess the implementation.

Rather than focusing on the issue of (3) *motivating users to contribute problems and discuss solutions*, we could instead focus on how users become involved in *A Specific Project*, thinking of these as typically *Minimum Viable Projects*. Minimal, first, in sense that they are often based on relatively small-scale, one-off experiments that will not need to be repeated once the project has run to completion. And second, in the basically microeconomic sense that users will only be attracted to projects which are seen to convey benefits in marginal utility. The issues of "trust" and "credit" that were highlighted in the initial focus group are two small but important facets of what might make a project viable. Users also need examples to follow that show how they can get involved. Question-answering – which comprised a key set of features that were intended to motivate users to contribute problems – addresses primarily informational needs. This sort of feature does little to address interactional needs, which robust support projects would accomplish. Groups on their own do not seem to provide sufficient structure. All of the dimensions of viable systems (Beer, 1981) should be given due consideration in further design and implementation work.

Finally, individual projects – including the software implementation project – can (4) *support mathematical meaning making* – but the details of just how this is supported would be made more clear using the *Frontend and Backend* pattern. In particular, *Frontend and Backend* is directly associated with (5) *support for behavior tracking and semantic modeling*. If PlanetMath users were more easily able to cross between the mathematical and computational ways of thinking, including work with data and analytics (as in Chapter 6), we would be well on our way to building an "open engine culture."

9.5 Summary

The layout of Figure 29 is reminiscent of the model of group informatics (Figure 5 [§4.3, p. 55]) from Goggins et al. (2013), but it adds more flexibility to describe the emergent category these earlier authors called "*contextualized interactions*." The approach provides a "meta-model" that looks for emergent order relative to given boundary conditions. As new structure forms, it becomes part of the boundary conditions for future iterations.

Figure 29 explicitly suggests an "open world" model in the sense that projects can be hooked together. The design pattern methodology offers a practical form for a theory of peer learning and peer production in distributed projects. Caroline Wagner (2008) highlights "*emergence*" as one of the key features behind the rise of a "*new invisible college*" among globally networked scientists. The approach seems to be a general and replicable method for doing ongoing emergent design in a peer produced peer learning context.

In the current setting, three design patterns, *Frontend and Backend*, *Spanning Set*, and *Minimum Viable Project* summarize the key take-away from user engagement with PlanetMath/Planetary. In particular, the "What's Next" sections of these patterns suggest practical steps that directly address the main research question [§1.2, p. 5]:

- At PlanetMath, we have an *open engine*, but not yet an *open engine culture*. Along with continued work to serve and understand specific users by focusing on specific and concrete use cases, we will want to build pathways for meaningful user involvement in work with the software system.
- As a project with an encyclopedic component, PlanetMath can be used to span and organize a significantly larger body of existing material. The high-level idea is that of a "cross-index" to the mathematics literature. Initial prototypes should be built for smaller domains (for example, Calculus or Mathematical Biology).
- The front page of the website could be redesigned so that the top-level view of the site is project focused. That is, instead of collecting all of the posts from across the site – or even all of the threads from across the site – the front page would collect succinct summary information on recently active projects.

10 Discussion

10.1 Recapitulation

I have argued that mathematics is a socially shared, symbolically – and technologically – mediated cultural activity. In light of this, I asked: Could learning mathematics become more like the learning that happens in the course of developing free/open source software? While theories of pedagogy can help us understand conditioning practices, they fail to model the productive, explicating, way in which learning unfolds in online peer production communities, which is described here with a theory of *paragogy*. The paragogy framework I introduced suggested a way to organize and survey prior work in adjacent fields, and formed an organizational tool useful across several scales throughout the work. The core research methodology aimed to build a "thick description" of mathematical social life in the form of a working system for peer produced peer learning. The first study focused on understanding the effects of peer support on learning in the course of encyclopedia authoring activities and forum discussions on the original PlanetMath, documenting the catalysis of learning outcomes. The second study employed qualitative methods to build a richer picture of the stakeholders in a peer learning environment for mathematics, using the paragogy framework to derive directions for change. Informed by this work, PlanetMath's software system was rebuilt along the lines specified by the paragogy model, augmenting the collaborative authoring environment with tools for peer supported problem solving. Qualitative findings from a user study after the public release of the new system demonstrate the feasibility of the paragogical approach, while suggesting a range of improvements to the software that will be needed for the project to successfully scale up – in particular, the further re-structuring of PlanetMath's activities into "sub-projects."

Context	Feedback	Quality	Structure	Heuristic
$A \leftarrow A$ $A \overset{\ell}{\leftarrow} A$ $X \hookrightarrow \mathbb{X}$	$X \leftarrow T$ $S \leftarrow R$ $\mathbb{X} \rightarrow \mathbb{X}^\sharp$	$X \leftarrow Q$ $A \leftarrow C$ $X \rightarrow X'$ $\mathbb{X} \vDash \mathbb{X}^\star$	$A \leftarrow P \leftarrow \mathbb{J} \leftarrow S$ $L \leftarrow A, P$ $M \leftarrow A$ $Q \leftarrow A$	$G \hookleftarrow U$ $S \hookleftarrow H$ $Q, T \rightharpoonup C, W, P$ $G \hookrightarrow \mathbb{E}$
A article ℓ link X object	T post S solution R review	Q question C correction	P problem L collection M classification	G group U user W request H heuristic
$\mathbb{X}$ project	$\sharp$ update	$'$ fork $\star$ outcome	$\mathbb{J}$ conjecture	$\mathbb{E}$ ephemera

Table 24: *Entity relation diagram for PlanetMath 3.0*

10.2 PlanetMath 3.0

Table 24 presents a revised version of Table 10 [§5.2, p. 74], which now includes the insights gained in the user evaluation of PlanetMath/Planetary discussed in Chapter 9. Any object, or activity, that takes place within the system should be thought of as being part of some project. This carries over to the "micro" level, so that individual proof steps or computations are to be thought of as small projects. In particular, one important class of "project" is a conjectural solution to a problem, which can generally be thought of as a (fallible) process for arriving at a solution. In the process of working on problems, groups would like to be able to form on an *ad hoc* basis and create ephemeral content, the online equivalent of a chalkboard that can be wiped clean. Articles – and presumably other forms of project-embedded content – should be forkable. Projects should be updatable, with revised goals or with concrete indications of progress.

Projects can be modeled both in terms of user experience and computational features. Project updates can be modeled with a language of fundamental actions. Projects themselves model their outcomes to some degree of fidelity, and such models provide an interface for other projects and for potential participants. Projects are made "viable" by features that connect to the motivations and ambitions of participants.

Chapter 6 showed results from instrumenting the corrections and forum posts and analysis aggregated data from PlanetMath's contributing population over a decade. It would be useful to instrument the entire system in order to be able to offer real-time feedback on learning relevance of interactions across the system. For example, a sufficiently instrumented system might be able to generate charts of progress on solving specific problems on the fly; *cf.* (Schoenfeld, 1987).

10.3 Channels for communication between the strata

As per Hill (2011a), it may be most convincing to market the rebooted PlanetMath project as an effort to build a familiar kind of resource in a new way: for example, to create a free/open clone of a familiar set of textbooks, like the Schuams outline series or the Springer Graduate Texts in Mathematics series, by assembling them out of smaller pieces. Another familiar mode of engagement is the mathematics course. Traditional courses could be run in 1-to-1 (or many-to-1) correspondence with the textbook series mentioned above. One can compare the earlier experiment conducted by Robert Milson, in which he used a local instance of Noösphere to run a graduate-level mathematics course (Milson & Krowne, 2005). However, it would be even more propitious if a course adopted PlanetMath itself as an interdisciplinary "learning project." How might this work? For comparison, consider William Thurston's reflections on "Mathematical education" (Thurston, 1990) and Frank Quinn's perspectives on "A science-of-learning approach to mathematics education" (Quinn, 2011).

For Thurston, an ideal teaching environment was a course on "*Geometry and the Imagination*" that was "*taught as a team, shunning lectures and emphasizing group discussions among students*," culminating in a "*geometry fair*." Quinn's ideal instructional context is "*over a thousand hours of one-on-one diagnostic work with students in the Math Emporium at Virginia Tech*", covering material "*designed so that pre-existing learning errors will cause serious difficulty*," with the intention "*to expose these errors so they can be diagnosed and repaired.*"

Appreciating one of these perspectives need not lead us to abandon the other. After all, "*understanding comes on many different levels*" (Jaffe & Quinn, 1994). PlanetMath may find new purpose in fulfilling the request voiced by Thurston (1990) for "*the creation of channels for communication between the strata*." Surely methods that work – and both Quinn and Thurston have strong points here – should be precisely what is shared and developed through such channels.

One of the participants in the initial focus group I ran at The Open University felt that PlanetMath would be more successful if it was more limited in scope.

> *I think that the planetmath project needs to be sharpened in terms of what it is trying to achieve. That may be done by narrowing down the range of users that it is directed to. Considering the fact that lots of financial resources are spent from formal institutions (like the OU) to develop online platforms for delivering their services, it might be quite hard for planetmath to compete with such platforms.*
>
> Anonymous study participant, excerpt from questionnaire response

It would be useful to sharpen the focus of the PlanetMath project, but not by limiting the range of users it serves, topics it covers, or features it offers.

Rather, PlanetMath needs to maintain a sharp focus on the goal of global mathematical literacy if it is to be successful in establishing effective communication between state school and Ivy League; between first and third world; between researchers, teachers, and students at all levels; and between many individual peer learning projects with their own specific focused scope. The project as a whole may meet with ambivalence from some parties with deeply embedded institutional commitments; *cf.* (Nietzsche, 1878, pp. 51–52). Even so: "[T]*he emergence of novelty requires that objects be at once both in the old system and in that which arises with the new*" (Mead, 1932).

10.4 Teaching mathematics the commons-based peer production way

> *To teach is to honor our commitments by repaying what we owe the world for our formation. In short, teaching (and not ethnographic writing) is the other side of participant observation: there cannot be one without the other, and both are indispensable to the practice of anthropology as an art of inquiry.* ***To teach anthropology is to practise anthropology; to practise anthropology is to teach it****.*
>
> (Ingold, 2013, p. 13), original emphasis

One of the outcomes of the Peeragogy project was a general-purpose framework for building peer learning projects (Corneli, Keune, Lyons, & Danoff, 2013). The Peeragogy project is currently using this syllabus as the basis of a new mutual-aid effort for free/open/peer produced projects called the Peeragogy Accelerator.* This framework can also be instantiated as a syllabus for participation in PlanetMath, as a design for "*active learning*"; *cf.* (Freeman et al., 2014). The idea would be to study mathematics and mathematical anthropology together, and in the process, build and share resources with others. Rather than a classroom culture [§1.3, p. 9], teachers and students who look at mathematics this way would find themselves part of a global culture – but it is one "*without any central committee that would have to, or even could, tell the active what their next operations should be*" (Sloterdijk, 2013, p. 402). This underscores a key refinement to the notion of paragogy introduced in Chapter 3 that has been emphasized in the design coming from Chapter 9, namely the need to focus on individual projects. The model in paragogy is not simply one of unbounded productive emergence, but always an emergence with limits. This is not made sufficiently clear by the five paragogy principles, which rely on the word "context" to indicate a sense of scope. It must be emphasized that even projects operating with a global scope always run with discretely bounded work cycles; *cf.* (Kauffman, 2001). The use of design patterns has helped to make it clear to work on focused local tasks and share the results globally. Design patterns form a key part of this plan for teaching mathematics the commons-based peer production way.

*http://commonsabundance.net/groups/peeragogy/

1. Setting the initial challenge and building a framework for accountability among participants. Identify a challenge that mathematics or computing could help you solve. How have people approached this sort of problem in the past? What role do technology and media play in previous approaches to this sort of problem? What's at stake now? Share your intention and refine it in discussion with others. Be prepared to go through several rounds of revision as you convert the initial idea into a manageable project. Again, you'll want to consider the way other related projects have been organized.

2. Other people can support you in achieving your goal and make the work more fun too. Create a project on PlanetMath and find join a few adjacent or related projects. One place to find of related projects is the list of previews of new features in PlanetMath's issue tracker.* Individual MSC classes can also be considered to be content-related projects. Look around on the internet for other relevant projects. What other problems are people working on that are related to the problem you have identified? Can you collaborate with them? What do you need to learn in order to be helpful in these projects? What help are you likely to need?

3. Solidifying your work plan and learning strategy together with concrete measures for 'success' can move the project forward significantly. Find or develop collections or sub-projects that specify the reading material, exercises, and prototypes that will help you move toward your goal. Arrange these into a project roadmap, and revise it as you go. Is anyone interested in your intermediate steps? If you run into difficulties with content or software communicate them to the relevant people. Figure out how you can get take part in resolving the difficulties, and share the patterns that work. If you get stuck, write down and share an antipattern describing the problem, and look for a suitable work-around.

4. Wrap up the project with a critical assessment of progress and directions for future work. What did you learn? What roadblocks or bottlenecks did you have to overcome? How will you change your approach when you tackle the next problem? Be sure to upload a project summary with pointers to any new code or content. Assessment is easier if it's ongoing: each of the subprojects you've created should be assessed, and some of them should be wrapped up before the main project is complete. Once you've finished a sufficiently large project, give yourself some time off before you start again on something new.

10.5 Limitations

This thesis has been restricted to developing the following three claims.

- A theory of peer produced peer learning was developed, which was given the name paragogy.
- A working model of a paragogical approach to mathematics learning has been developed and deployed as PlanetMath/Planetary.
- A user evaluation of the deployed system has been conducted using principles of emergent design, which has resulted in refinements to both model and theory.

Apart from this clearly circumscribed focus, there are other limitations.

The clearest limitation of this work is that it does not include a classroom study or any other longitudinal analysis of learning under the paragogical model. Accordingly, we cannot say: "prior to the introduction of PlanetMath/Planetary and the paragogical approach, student achievement on such-and-such standardized test was at this level, and now it has shifted on average by Y%." Uptake of the new tools on PlanetMath (Figure 1 [§1.5, p. 12]) has not been sufficient to warrant a study of learning using the methods of Chapter 6. From the point of view of studies of this nature, the work presented here is a necessary preliminary.

Planetary is already used as a course support tool – but evaluating the paragogical model in a mathematics learning context rather than a design context would mean running a course like the one outlined in Section 10.4, and for that, more quite a bit more groundwork is needed. Some form of longitudinal evaluation would be important for developing any policy considerations that might follow from the idea of paragogy.

Although conducting a classroom based evaluation seemed, at times, to be almost within reach, simulation work artificial agents has remained entirely out of scope. Many of the ideas discussed in the thesis may be germane to such studies, but this claim has been in no way tested. This means that even though the thesis points toward a computational model of mathematical collaboration, the usefulness of this model remains conjectural; and its empirical validity stands, for now, only on its own apparent self-consistency.

Inviting PlanetMath users to become familiar with the pattern catalog and write new patterns would have been an interesting opportunity to more fully involve participants in co-design, and to learn more about whether design patterns are indeed suitable for use in the mathematics domain. However, when the studies ran, the pattern catalog was still in a relatively formative state, and would not have been as easy to share or

explain as it is now. My methods evolved along with the research project; *cf.* (Tom, 1996). The Peeragogy community provided a "*structurally parallel situation*" (Cavallo, 2000B, p. 141) in which to develop many of the design ideas. However, the engagement between Peeragogy and PlanetMath and has so far remained mostly "virtual" – and as yet there has been no formal study *evaluating* the pattern methods as employed in either setting.

Spending time working on the Peeragogy project and, earlier, running courses at P2PU, gives a range of experiences to compare with, and, accordingly, helps to make the work theoretically robust. However, exploration comes with costs, and in particular, empirical work trades off against hours spent programming. There were certainly many hours spent on the programming effort, along with many trips to meet with collaborators at Jacobs University. Nevertheless, my initial hope to build actionable semantic domain and user models, as represented by the use case in Figure 3 [§2.3, p. 31] was not realized. Much of the relevant *basic* infrastructure now exists, but recommendations and automatic tutoring have to be left for future work.

Regarding recommendations and diagnostics, the model employed in Chapter 6 has yet to be extensively tested in limiting cases, for example, to ascertain how well it works on small data sets. The model does not yet include confidence intervals. The method would have to be modified further to work in real time as part of a recommendation system. Work with simulated learners would be one route to testing and improving the model, *e.g.* informed by (McCalla & Champaign, 2013).

To ground paragogy as a scientific theory and not just an "approach," it will be necessary to make it falsifiable. This may be most easily achieved by working with a collection of paragogical "micro-theories" presumably based on collections of design patterns (Corneli, 2014), but the precise method for doing the testing is left for future work. Without further development at this level, and without further empirical studies of problem solving, the idea that the work discussed here could add insights into social problem solving beyond those developed, for example, in (Pease & Martin, 2012), remains conjectural – although it does offer a new approach.

To summarize: the primary methods employed in this thesis were statistical analysis of historical data, qualitative field work, implementation of a software system for collaboration, and emergent design. As applied, each of these has certain limitations which might be addressed through further work. However, the greatest limitations might be addressed with methods that were not employed in the thesis, in particular methods coming from artificial intelligence. These were explicitly set aside in order to pursue what I see as groundwork. I genuinely hope that the ideas will be taken forward by others. I outline my own planned contributions below.

10.6 Future work

In computer science: In order to build a computational study following from this work, one would need: a reasonably large computer-accessible body of mathematical content (perhaps along the lines of my earlier work in mathematical knowledge representation [§2.1, p. 22]), along with computational agents that are able to navigate the relevant mathematical structures (as outlined Table 24), able to apply "standard" (Table 4 [§4.2, p. 53]) and "social" problem solving heuristics (Table 13 [§5.2, p. 79]), sufficiently metacognitively aware [§1.2, p. 9] as to be able to set and solve problems by generating and applying new heuristics in the form of design patterns (Table 11 [§5.2, p. 76]) – and ideally able to annotate, reflect on, diagnose, and extend the overall process (see Appendix F). Appendix B contains two examples that have been worked out in a preliminary manner – but more work is needed in order to fully describe a mathematical proof or exposition as a social-computational processes. To further develop examples of this nature, it would be useful to thoroughly reassess the five dimensions of change described in Chapter 3 – for example, are they really the right ones for dealing with mathematics at the micro level? – and to develop a catalog of design patterns expressly devoted to proofs, problem solving (Posamentier & Schulz, 1996), and problem posing (Brown & Walter, 2004). In particular, the notion of problem posing gets at the essence of pattern-making: we will need patterns about patterns in order to obtain the kind of closure alluded to in the introductory comments on the pattern analysis in Chapter 9 [§9.3, p. 149]. The plan would then be to implement a system that can work with these patterns using a society-of-mind style architecture (Minsky, 1988). Related work using artificial agents in a narrative reasoning context was discussed in (Singh & Minsky, 2005), with a detailed development in (Singh, 2005), drawing on the earlier notion a computational critic (Sussman, 1973). This work emphasized reflexive aspects of mentation and multiple ways to think about problems (Minsky, 2007).

In humanistic computing: PlanetMath should at least provide a more satisfactory learning platform for beginning mathematics students than Wikipedia. The idea that peer supported online learning might also be "better" than standard classroom participation, in the sense of Bloom's 2-sigma problem,* does not require a great stretch of the imagination. What it does require is the opportunity to carry out the necessary research.† David Smith (2002) writes that "*learning mathematics is, first of all, learning,*

*Students tutored one-to-one were observed to perform *two standard deviations better* than students who received conventional instruction; Bloom (1984) asks if there are methods of group instruction that can be as effective as individual tutoring.

†As motivation, one can compare results from studies of the JUMP math project from Mary Jane Moreau, and Tracy Solomon et al. (forthcoming); see (Mighton, 2014) and the overview in "A Better Way to Teach Math", David Bornstein, April 18, 2011, The New York Times, HTTP://OPINIONATOR.BLOGS.NYTIMES.COM/2011/04/18/A-BETTER-WAY-TO-TEACH-MATH/

and only secondarily about mathematics." It is not necessary to wait for the results of further studies of learning on PlanetMath to pursue related work in other sectors (Corneli et al., 2014).

Focusing on content: PlanetMath now hosts, on a demonstration basis, a copy of a recently published free/open textbook on Homotopy Type Theory (Univalent Foundations Program, 2013), and a retrodigitized 20th century calculus book (Davis & Brenke, 1912) both of which have been assembled using interconnected collections.* The sections of these book are discussable, the problems from the book are available to be solved, and some solutions have been added. The contents are presented in MathML, which brings with it the possibility of deep linking and expression-level annotations. The reader's interface could certainly be improved, and integration with Git should be added – after the manner of the deployment of Planetary on MathHub.info (Iancu et al., 2014) and as preferred by the original authors of the HoTT textbook.† And, of course, more books should added as well. Digital libraries are an active area of interest for the mathematics community (National Research Council, 2014). Planetary may prove to be quite useful in this distributed effort, since the system can be straightforwardly adapted to support disparate use cases. Naturally, the recent National Research Council report focuses on research-level applications, but as outlined in Section 10.3, PlanetMath can work on making this effort relevant to stakeholders and constituencies they see as secondary, including "*students below the advanced graduate student level*" and "*researchers outside of mathematics*" (National Research Council, 2014). The National Research Council report expresses disinterest in the large-scale retrodigitization of public domain materials – but, even so, considerable progress could can be made with off-the-shelf OCR tools and small amounts of funding to support a crowdsourcing approach. David Cavallo's focus on finding "*bridges to new content*" (Cavallo, 2000a) is particularly interesting at large scales. Rather than viewing Wikipedia, math.stackexchange.com, and MathOverflow as competitors, they could seen as potential collaborators and contributors to the same commons resource (they all use the same CC-By-SA license as PlanetMath). Perhaps PlanetMath's best application at the moment is not to generate new content, but to focus on organizing and integrating content from these sources and from the public domain.‡ Finally, having laid considerable groundwork for the use case outlined in Figure 3 [§2.3, p. 31] both in this thesis and in related work (Mulholland et al., 2013), it is worthwhile to finish the job and add a suitable recommender system to PlanetMath, particularly in connection with this content-focused work.

*http://planetmath.org/users/pmbookproject

†https://github.com/HoTT

‡http://meta.wikimedia.org/wiki/Grants:IEG/PlanetMath_Books_Project

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A DIY Math: Course description and post mortem

COURSE TITLE:

DIY Math

COURSE TWEET (200 CHARACTERS OR LESS):

This course is designed to build independent study and peer-support skills for mathematics learners at all levels. It will require both self-directedness and active participation.

FACILITATOR:

Joe Corneli (Board Member at PlanetMath.org; PhD student at the Knowledge Media Institute, The Open University, UK; `HTTP://METAMESO.ORG/~JOE`)

COURSE DESCRIPTION (NO MORE THAN 500 WORDS):

This is a mathematics course open to all topics and levels (unless participants come up with a compelling reason to focus during the course). The main "pedagogical" reasons for such radical openness are:

(1) To give people one extra reason to take initiative in their own learning. One might say "I've always wanted to learn about *P* vs *NP*", another might say "I've always wanted to learn precalculus" – both are overcoming a mental and emotional hurdle. I'm not suggesting that it's the *same* hurdle, but there are probably some similarities. In the same way that a writer's workshop can serve writers with different interests, this "DIY Math" workshop can serve mathematics learners with different interests by providing a place to talk about their challenges and successes.

(2) To help people develop skills at co- or peer-to-peer teaching. Ultimately people have to do their own homework exercises and so on, but one can also learn a lot in mathemat-

ics by helping others. My "teaching philosophy" is that there are no stupid questions, even if there are a lot of dumb answers. Let's see if we can get better at answering questions together.

There is no official text book, but participants may want to take a look at "How to solve it" (or other books) by George Polya. Participants will get access to a new "beta" grade libre software platform being developed to support mathematical communication; we'll also have a mailing list available as a fall-back mechanism (and if you're interested in the course, please sign up for the list now to talk about your ideas or any questions you have about the course).

PREREQUISITES:

The willingness to learn. Also, you'll need to come up with some realistic goals for a 6-week mathematics course!

AUDIENCE:

I hope the participants will want to both *ask* and *answer* questions. Collectively I hope that they will have varied backgrounds and aspirations.

EXPERIENCE:

I have a bachelor's degree in mathematics from New College of Florida (2002), and I did a couple years of math grad school before switching to a more "DIY" approach. My current PhD project has to do with "Crowdsourcing a Personal Learning Environment for Mathematics" and this course can be considered to be an informal dry run.

POST MORTEM:

Dear DIY Math participants:

I feel like this course is effectively over (and maybe was even "over before it began," though I think none of us saw that). Rather than trying to squeeze more out of the course than it has to offer, I'd like to share my reflections on "what went wrong."

This mailing list, of course, continues to be a place to discuss "DIY Math" in general, and perhaps the course will rise again sometime. I'd like that, and I am not discouraged by what I've learned so far. Further, I wish to salute all of you for your efforts to be self-directed learners. But I think we need a different approach.

The following is my own post-mortem analysis of the course. If you would like to add to it please do. If you feel like the course is not actually over, now would be a good time to speak up about that, too.

My assessment of the course was that participants knew *more or less* what they wanted to focus on, but they didn't exactly know "how" to start, and they didn't particularly want to discuss questions about "how to do things."

Unfortunately, "how questions" were supposed to be the central focus of this *discussion-based* "DIY" course.

At this point, I can see several profound shortcomings in the way I designed the course, not the least being that the implied tension between "doing it yourself" and "discussing it with others" was not given enough attention up front. I had hoped this would be a productive tension or "dialectic," but it doesn't seem to have turned out that way. There is a certain degree of commitment required to work with(in) such a dialectic, and no one who signed up for the course seemed particularly interested in that. Rather, applicants for the most part "just wanted to learn math." I did not do a very good job explaining why I think working with the diy/p2p dialectic is *how* to learn. Moreover, my personal philosophies aside, I simply didn't do enough to get people talking in the course, partly because I rather blindly assumed that the desire to discuss would be shared by all participants. Or, more precisely, I assumed that those who wanted to discuss the most would carry the conversation forward.

Aside from these issues, which I take responsibility for, some of the shortcomings of the course itself point to design considerations that would best apply at a higher level; something like "P2PU best practices." One could say that course-level and P2PU-level issues have some degree of "trade-off" between them.

As one very simple example: If P2PU was taking an ongoing survey of the "wished for" course topics (to be tagged and voted up or down by site visitors), I would presumably have been able to create a mathematics course tailored to the interests of pre-self-selected participants, instead of fashioning a course that was supposed to serve all mathematical interests simultaneously.

As it turned out, I learned at the beginning of the DIY Math course that the interests of the applicants were approximately equally distributed between (1) General interest, e.g. learning how to learn mathematics; (2) Technical interest, e.g mathematics for computer programming; (3) Mainstream mathematics, e.g. precalculus and calculus; and (4) Advanced mathematics, e.g. probability and statistics.

One obvious thing to do "next time" would be to simply run a "standard" mathematics course for each of these four segments. This would be a lot of work to set up, but by no means impossible: perhaps I could find some other people who would want to co-facilitate these courses. My guess is that with significant work and a design that draws on P2PU's collective experience of how to teach (and how *not* to teach) "academic subjects" in a peer-to-peer setting, these courses would all be reasonably successful.

I'll also note that it would be possible to focus on "how questions" in any of these courses (perhaps especially obvious in the case of the first group, since one could copy an existing course on mathematical foundations). Nevertheless, I think the course would most likely need *a lot* more structure in one form or another. There are undoubtedly several different ways to go about structuring something like this, but my guess is that *either* we'd need a more step-by-step curriculum crafted to ease people into thinking about "how they learn," *or else* we'd need to know in advance that participants were really committed to asking and answering "how" questions.

Regarding this issue of "commitment" – which in fact seems quite important independent of the course's "peertagogic" style – in the latest P2PU community call, Alison Jean Cole again reminded me about the idea of setting up a *social contract* for the course.

I said to her that this seemed like another issue that could potentially benefit from support coming from a level above individual courses, e.g. by involving an "course mentor" who would help facilitate the creation of a social contract for the course. This could be someone who had previously run a "successful" course.

Such an idea might go along with re-thinking and nurturing the "social contracts" implicit to the P2PU structure itself (*i.e.* we would ask ourselves what we share as a broader learning community). Some other grassroots organizing styles are very strong on this aspect, both those that are "radicalized" and others that are more politically moderate.

To sum up: I think a course built around "how to learn mathematics" is a great idea. Trying to involve mathematics learners at all levels in discussing this question is also probably a good idea, but a course may not be the best way to have such a discussion. Certainly people need to know what they are getting into in any case.

To conclude: Hopefully we can find the "right" group to make progress on these issues in the future. This course has brought up some of the core problems that apply to peertagogy. We should cut ourselves some slack, because this field is so new that I had to come up with a neologism here and now to describe it! If that's to be a lasting positive outcome of the course, I suppose we should offer a more formal version, "paragogy" (not to be too heavily confused with the etymologically related "paragoge"). Whatever we decide to call it, the key point of DIY Math as a genre is to do away with the artificial distinction between pedagogues and learners. This is a power struggle par excellence, easy enough to mock perhaps, but one I think we should not ignore.

Thank you for your attention,
Joe

B A paragogy-inspired coding for two mathematical proofs

This appendix applies the paragogy framework to a few short mathematical texts, to help show how the ideas about peer learning developed in the early sections of this thesis relate to the mathematics domain.

As example texts, I've chosen two mathematical proofs where "the obvious thing works", as presented by Timothy Gowers in a recent talk on *Modelling the mathematical discovery process* (Gowers & Ganesalingam, 2012). The detailed small steps presented in this talk give concrete examples of Gowers's notion of the "quantum of progress" as described in Chapter 4. The individual steps are coded as (1), (2), (3), (4), or (5), by matching into this scheme:

	Nowak	*Paragogy*	*Lakatos*	*Peirce*
(1)	kin selection	context	concepts	–
(2)	direct reciprocity	feedback	examples	induction
(3)	indirect reciprocity	quality	conjectures	abduction
(4)	spatial selection	structure	proof	deduction
(5)	group selection	heuristic	other	–
	"social theories"		"logical theories"	

This discussion can be connected to the Minskian heuristics for problem solving and the design patterns that were introduced in Chapter 4 and Chapter 5. This leads to the short outline of heuristics for social problem solving, which appears at the end of the appendix. This can be taken as a very preliminary sketch of potential "implementation details" for an agent-based system that would use design patterns to do proofs or tutoring in mathematics.

To prove: A compact subset of a Hausdorff topological space is closed.

First: maybe it isn't so important to know the meaning of the terms.

[(5) - This is an off-handed remark that is almost certainly not true, but it does suggest an important heuristic: restricting the unknown to whatever it is that we *do* know.]

Second: This is not true in non-Hausdorff topological spaces.

[(1) - This looks like an aside, or potentially an extra lemma to prove; however, the purpose it serves in this context is to ensure that we have the *concepts* right.]

The first step is slightly non-obvious. We'll try to prove that A^C is open.

[(1), (4) - This is an equivalent definition of the problem statement, however, it's "slightly non-obvious" why this is being used as a proof step. Later in the proof, we'll see why – for an expert who has a feel for the structure of proofs like this, this choice of phrasing is relatively obvious.]

So, given a point $y \notin A$, we need an ϵ-ball around y that doesn't intersect A.

[(3) - The idea will be to map this hypothetical y to some conjectural but concrete feature, the open ball that doesn't intersect A.]

Take a cue. You have a library of things that can be used in the proof. Use an equivalent statement.

[(5) - Again, we're reminded to restrict the unknown to what we know.]

Let $y \notin A$.

[(2) - This introduces the example we need to operationalize our main conjecture so far.]

Now expand something else. Because of **compactness**, *every open cover of A has a finite subcover.*

[(1) - Mechanical expansion of the definition.]

Finding an **open cover** *of A would mean finding a collection of sets $\cup$ such that $\forall x \in A \Rightarrow \exists U(x) \in \cup : x \in U(x)$, and U open.*

[(1) - Mechanical expansion of the definition.]

Because of the **Hausdorff property**, *$\forall u \neq v$ are such that there are disjoint open sets U and V such that $u \in U$, $v \in V$.*

[(1) - Mechanical expansion of the definition.]

We now know that "$U(x, y)$" containing x and not y exists because of the Hausdorff property (and because $x \neq y$). Symmetrically for $V(x, y)$. Let $\cup$ be the smallest set of

$\{U(x,y) \mid x \in A\}$. *Similarly, let* $\mathbb{V} = \{V(x,y) \mid x \in A\}$.

[(3), (2) - We're mapping the hypothetical y to some additional constructs, $\mathbb{U}$ and $\mathbb{V}$, building a more complete model of the space. As part of this complex gesture, the placeholder $x \in A$ is introduced as a example.]

Let $\mathbb{F} = \{U(x_1,y), \ldots, U(x_n,y)\}$ *be a finite subcover of* $\mathbb{U}$.

[(4) - We now apply the definition we expanded earlier.]

$w \in A$ *implies* $w \in U(x_i,y)$ *and* $w \notin V(x_i,y)$.

[(4), (2) - Applying previous definitions to move around the unknowns.]

$y \in V \subset \bigcap_{i=1}^{N} V(x_i,y)$. *This would be pulled out of the library.*

[(1), (4) - The fact that finite intersections of open sets are open is a key "concept" that can be applied. We have now positioned the unknown y in an open set that does not intersect A. There is also a bit of deduction implied.]

(All the way through, everything we did was extremely routine.)

[(5) - This certifies the proof.]

A 'Magic Leap' problem: compute the 500th digit of $(\sqrt{2}+\sqrt{3})^{2012}$**.**

What is the 500^{th} *digit of* $(\sqrt{2}+\sqrt{3})^{2012}$*?*

[(1) - This is just the statement of the problem.]

Even this, eventually, a computer will be able to solve.

[(5) - This is an aside, but it points out that there is something here that is non-obvious – *i.e.* an "unknown".]

For now, notice that total stuckness can make you do desperate things. Furthermore, knowing the origin of the problem suggests good things to try. The fact that it is set as a problem is a huge clue.

[(3), (1) - We initially guess that we're "totally stuck", but then think more about the context, trying to move beyond the page to see what might be implied. Note that the *social* context is referred to, rather than the *semantic* context.]

Can we do this for $(x+y)$*? For e? Rationals with small denominator?*

[(5) - We have now made a critical step, moving from a "known" blob of text to a statement with a useful unknown in it. We can then fill in things that we have some hope of doing calculations about in place of this unknown – for instance, $(1/2)^{2012}$ or $(1/10)^{2012}$.]

And how about small perturbations of these? Maybe it is close to a rational?

[(3) - We now begin to form a conjecture.]

m^{th} digit of $(\sqrt{2}+\sqrt{3})^n$?

[(2) - Introduce variables as a generalization of the problem statement. (We can't deal with the conjecture directly, so we'll explore.)]

$(\sqrt{2}+\sqrt{3})^2$*?*

[(2) - Try a particular example. We're now exploring the generalization of the problem.]

$(2+2\sqrt{2}\sqrt{3}+3)$

[(4) - Just computation.]

$(\sqrt{2}+\sqrt{3})^2+(\sqrt{3}-\sqrt{2})^2=10$

[(2), (3) - A somewhat clever idea: we've built an example that we can understand. However, this step is also abductive – we may have found a class of expressions that we can understand in a similar way.]

$(\sqrt{2}+\sqrt{3})^{2012}+(\sqrt{3}-\sqrt{2})^{2012}$ *is an integer!*

[(4) - All the terms with odd powers in the second product also have odd exponents on the factor −1, so they cancel with corresponding terms from the first product.]

And $(\sqrt{3}-\sqrt{2})^{2012}$ *is a very small number. Maybe the final answer is "9"?*

[(3) - We refine our earlier conjecture.]

We need to check whether it's small enough. $(\sqrt{3}-\sqrt{2})^{2012}<(\frac{1}{2})^{2012}=((\frac{1}{2})^4)^{503}=(\frac{1}{16})^{503}<.1^{503}$*, so we're in luck...*

[(4) - This is important to check! (In fact, we could show that $(\sqrt{3}-\sqrt{2})^{2012}$ is still smaller, but it is not infinitely small.)]

The answer is indeed 9.

[(5) - We've confirmed that $(\sqrt{2}+\sqrt{3})^{2012}$ is very close to, but just less than, an integer (by the right about)].

The following page collects the key ideas developed above and connects them with Table 4 [§4.2, p. 53] and Table 13 [§5.2, p. 79]. One can also compare the system design overview from Chapter 8 [§8.2, p. 113], which offers a very different but complementary interpretation of what are at their root the same five dimensions.

(1) One of the things that "*experts do*" is *expand the definition.* Experts also have the ability to appropriately "scope" the problem, to explain why it is feasible or important, and to convey some sense of purpose to others. An expert can find the relevant related concepts, establishing the context of the problem. In some cases, technical definitions can simply be "*pulled out of the library*" – but an expert knows which portion of the library to search in.

(2) "*Polling for ideas*" can help generate a set of examples. If you don't know quite what to do, you can try an example to generate ideas by considering a special case or a particular instance of the problem. In this way, in exchange for modest effort, examples offer feedback on the problem solving process. Pacing oneself and cultivating appropriate diversity among instances comprise an important class of heuristics related to example selection. Within a proof, variables can be introduced as examples, which can have either "global" or "local" significance. In some cases, finding even one relatively comprehensible example can suggest a theory that leads to the solution.

(3) Conjecture has to do with outlining a plan, sighting a path, or "*creating a guide.*" Concerning textbook problems, as "*The fact that it is set as a problem is a huge clue.*" Constructions provide analogies, and tentative explanations provide models. These models can subsequently be evaluated on their fitness-for-purpose. Such evaluations allow a problem solver to progressively build and refine an argument. Helpful conjectures are "rewarded" with further attention – non-helpful conjectures (e.g. "*total stuckness*") are abandoned.

(4) "*Changing the description*" of a construct – by symbol swapping and by using suitable definitions – can help shed to light on a problem. A new definition provides a concise "*wrapper*" for an idea or method. Deductions or proof steps are typically meant to be straightforward applications of the definitions. Although the structure of a proof reads "linearly", the way a problem solver finds the right proof steps is often nonlinear.

(5) A "*roadmap*" is a shared strategic plan that is allowed to change. It often comes with rules about the group of agents who are, in fact, allowed to change it. When formulating a proof, we often begin by restricting ourselves to the parts that we know – leaving little archipelagos of the unknown on our map, which can then be divided up and examined. The relevant operations in proofs are not simply about systematically breaking down the unknown, but also include the skillful introduction of unknowns.

C Selected quotes from initial fieldwork with stakeholders

Sections C1–C5 contain portions of dialog from the initial focus group with computing post-graduates that is described in Chapter 7. Discussants were **M1**, **M2**, and **M3** (self-identified as "more mathematical") and **L1**, **L2**, and **L3** ("less mathematical"). Sections C6–C15 briefly summarize the key features (setting, speaker, etc.) of comments by professionals that inform the later sections of Chapter 7. Primary discussants were Murdoch James Gabbay (**MJG**) and Alison Pease (**AP**). These discussions were not recorded; brief samples from my notes are presented in Appendix E.

C1 Dialog: Would you find this sort of tool suitable for use in your work or studies?

M2. *OK, so, yeah, the first thing that comes to mind is why would somebody would look for something like that? Especially if you assume they're not aware of this PlanetMath website, right. So this should be an ideal first place for someone to go, online, and look for some help. So, in my case, I'm studying, part time, a degree in maths here, at the Open University, and this – it's an online distance lecture thing – and therefor they provide the platform, you can have some kind of interaction with other students and stuff. So, like, the first point of reference, I would, like, go, in case I encounter problems – so, I'm working through the book, I'm trying to understand a method in Calculus, I have a problem, and I don't understand this – so I go to the forum to see if someone else had the very same problem and see if there's an explanation about that problem, and that would be the first thing I do. And if I don't get an answer there, I'll try to post something on that forum, which is already there. So, all I'm trying to say is that the way I open up is also related to the things I know. So, I wouldn't, you know, go online to look for a forum that I don't know, it might take some*

time also to familiarize myself also with the whole structure and how it works. and, you know, you need, like, when you concentrate, you're trying to understand something, you need to be, like, on a roll.

M1. *But there's one extra point, because if you're having, say, a problem with Calculus, would you trust the forum? So, do the users have a rating in terms of experience?*

M2. *Well, you have the – how do you call, it not regulators.*

L3. *Moderators.*

M2. *Moderators, yeah.*

M1. *But, we didn't mention anything about moderators or central control. Because, say, in Wikipedia or in other wikis you have some sort of moderators and central control just to ensure some sort of quality. Or can they just go and define something and then wait for others to say, well, this is wrong?*

L1. *Like, I think the discussion about, like, a course forum versus a more general PlanetMath type forum is an interesting thing to think about, um, because what I would think about, again, not from a math background, is kind of, instructor buy in, I think, like, the benefit of you going to your course forum is that your instructor is there, and your classmates are there, and you're using the same texts. And, again, I'm not knowledgeable about math, but, say if I were to plug in something about long division, there are different ways of teaching and learning long division, so I guess – the "pro" of going to something within the course is that you'll be taught, and you'll know, you'll have similar parameters or similar methodology. And whereas with PlanetMath, I think if I were going there as someone who is a total math newbie, it would almost overwhelming to parse the type of information.*

C2 Dialog: Describe your learning style

L3. *If I approach this thing from, I want to increase my mathematics skills, I don't exactly know where I am, because I stopped doing school courses 10 years ago, so I've gained some and I've lost some. I'm interested in this, and that, and generally very interested, maybe not in this, but this – but I don't search specifically for a topic, but just something that will get me further. And getting me further sort of implies that it's at my skill level, so that I can understand the text. So that requires a little bit of searching around, probably looking at different things. What really would would help me is trust, again, trust in the fact that if I don't understand something, or I'm stuck, it's my own fault, it's not due to an error in the problem or article. So that's where I would start, and then, presumably it would be nice to have something like a map or a histogram or a list or a chart of the area – the charted area, or the uncharted.* [...]

L3. *What is it like, you get a question, the square root of 3 for example, and then you answer it – if the question's more complex, what does the answering look like, do you sort of go away, and do your scribblings on some paper, and you find the answer and you post it in, or is it more like, is the problem solving, progressive, in steps?* [...]

L3. *So I think that for people like* **L2** *and me who like to do prototyping, and do iterative solutions, and approximate a solution* [...]

C3 Dialog: How do you currently solve problems?

M3. *It's not practical for logic, but when you do some programming, you have to parse some text, you would look if someone else had solved it and use their solution, just to save some time.*

M2. *I think what's important is the immediacy of trying to solve it. You're there, you're concentrating. You don't want to lose this concentration, get familiar with a website, that would totally destroy my concentration. The first thing I would do is stick with it.* [...] *And it could be, like, an exercise that you are trying to solve for the assignment that you have to submit, and you wake up with the solution in your mind. So that's one way of solving the problem. Then, you might need to go to the forum and see if someone else had the same problem and – sometimes there are students who have the same problem, and there are a lot of students and they reply and they get it right. Or maybe the moderator comes in the end and says this is how it works and maybe you want to look at that.*

M1. *It depends on what you're using the forum for – I use both strategies depending on the problem. Sometimes you just get a generic error message and it's not really clear and you just have the message on that. So you go to Google and you find dozens of posts on forums that will give you the solution almost instantly. But sometimes if you're trying to develop something that's a bit complex, that's not the problem, you're just developing it and it takes time. And you need to just work at it.*

L1. *To me it depends a little bit on where I am on in my own lifecycle of research. Like, when I'm starting out, and I'm doing more general thinking like – again, this is not a math example, but say like which theorists I need and are kind of essential for foundational knowledge – I think what I would do to solve that problem is different from what would do when I'm deep into my topic, and I then I do more, kind of, introspective problem solving, or I'll email a former trusted adviser, who I know kind of shares my orientation – I think I refer more to personal sources when I'm more deep in it, versus when I'm doing more general, then I'll turn to forums or do Google search. I think it depends a little on stage of problem solving. Or, maybe, also the kind of sophistication of problem solving.*

L2. *Yeah, I also think that it depends on the topic.* [...] *Motivation is important. Looking*

at examples – then again, it depends on the person, it's quite general.

L1. *It always depends on the type of problem that you run into – is it a boring problem that I just want to get out of the way, is it a personally interesting one, and you want to play with it.* [...] *Do I expect to get something out of it, or do I just want to get it out of the way?* [...] *There are also formulated problems that come to you in a formulated way, like an error message for example, or problems that you have to first formulate. And also, old problems that people have had before, or new problems that nobody has had before.*

C4 Dialog: Describe your learning style (continued)

L2. *To me, with the learning style, I like the learning by doing, and I have to say, with maths, usually there is this problem, that usually they teach without getting hands on, and for people who learn by doing, sometimes that's kind of an issue.*

L3. *How important would you say immediate feedback is?*

L2. *Yeah, visualizations, or...*

M3. *Can I just say one thing about learning styles? I think it's not just important for you to know everything, but it's very important to know how to find things when you need – I think that's more important, rather than trying to understand everything. I mean, in general. I think the skill would be nice – you cannot kind of capture, understand everything in all domains, it's too ambitious.*

L2. *The platform, also, in this case can play a role, right, of making it easy for you to find problems.* [...]

L3. [When compared with workbook style problems] *that's a way more interesting problem, of how do you scaffold complex problem solving. I mean, one easy approach would be "making aware of possible approaches" – can you break the problem down into subproblems, can you do a sort of root-cause analysis (I don't know if that's used in maths, but other areas certainly). Just probing the... prompting the user with different ways to think about the problem, if it's not trivial.* [...] *Can you use substitution? Made me think about, can you build an ontology of problems, make an estimate of the size of problems in terms of the number of steps usually required.*

C5 Dialog: How do you deal with people whose viewpoints are very different from yours?

M2. *I was thinking about the learning styles – yeah – what kind of learning? You have opportunities to learn different things in different situations, right. And normally you learn something after you have defined certain aims. So in my case, I want to understand the*

mathematical theory behind the computational model I'm using, and therefor I decided I have to build the mathematical background. And how am I going to do that? I'm going to build it from from scratch. So, you know, I defined my aim, how am I going to learn something. And that's one way to do it, I'm going to follow it, I'm just doing it there. You have other aims, like, sometimes, you realize, oh my god, you're having a conversation, and you realize... like, this guy learns this way, and I'm not really doing this. Like, for example with **L3** *yesterday, he was – we were having a discussion, and he was telling me, you interact with other people and this is how you learn. And if I think of myself, and me doing the PhD, over the past how many, 3 years, I haven't really done this to, um, a similar extent that* **L3** *does it, I mean,* **L3** *does this as a practice – I mean, I'm interacting with my supervisors, and I have this kind of thing, but you have much more opportunities to get out there to see how people learn, and bla bla bla. So, that also gets at the other thing, which is, how do you deal with people whose viewpoints are very different from yours? So, I think the different viewpoints has to do with the aims that people have in mind. They, basically, they use a certain kind of learning that they have identified as a proper one, in terms of getting somewhere, that, there is an aim behind it. And therefor, you see, OK, these people have different aims and different learning styles for achieving these aims...*

M1. *Yeah, sorry, it might also be a misconception so, hence, sometimes it's a different viewpoint because it's just a different point...*

M2. *OK, yeah, yeah, how, with regard to a particular...*

M1. *Problem let's say, and they would like to solve the problem this way, but they've got to conform to the standards.*

C6 Comments on online mathematics

"*It would be useful to have better systems for giving and receiving feedback.*"

MJG, comments after my talk presented to the Mathematical Reasoning Group at the University of Edinburgh, and reemphasized in our interview at Heriot-Watt. **MJG** told a story about how he had tried to learn about ultrafilters from various sources around the web, some that were very incomplete, and some that included outright mistakes. Generally, these sources do not come with easy ways to comment.

C7 Comments on encyclopedia articles

"*Highly-abstracted and non-expository articles are hard to understand: they suffer of impenetrable quasi-formality.*"

MJG, discussing ultrafilters (Figure 11 [§7.3, p. 97]), *etc.*, in our interview.

C8 Comments on student attitudes toward peer learning

"*Students are not comfortable sharing their work with other students. They are only somewhat comfortable showing it to their professor, whom they regard as one might a medical doctor.*"

MJG, commenting on his experiences with "peer pedagogies" in our interview.

C9 Comments on computational tools for mathematicians

"*Formalisms are only useful when the model can't be held in your head.*"

MJG, in the discussion following my talk at University of Edinburgh.

C10 Comparison of PlanetMath and Wikipedia as reference resources

"*Wikipedia seems to have better coverage, better exposition, and a nicer-looking interface.*"

MJG, discussion in our interview, based on a side-by-side comparison of the two websites.

C11 Alison's description of her approach to research

"*Empirical studies of mathematical discourse are a key research focus.*"

AP, first interview, describing her approach to research. Further details appear in Table 15 [§7.4, p. 100].

C12 Alison's description of her approach to research (continued)

"*How does experience build towards understanding?*"

AP, first interview, discussing themes in her research.

C13 Tools that could be useful for studying mathematical discourse

"*It would be useful to be able to detect the first use of a word within a given episode of discourse, and to measure it's impact.*"

AP, second interview, looking at how PlanetMath could be directly helpful in her research.

C14 Comments on computational tools for mathematicians

"*Can we build more detailed models of mathematical discourse?*"

Informal discussion with Mohan Ganesalingam and Sir Timothy Gowers, of Cambridge University, visitors to the Mathematical Reasoning Group at University of Edinburgh, who spoke with me about linguistic modeling of the mathematics domain, and interactive software.

C15 Comments on artificial intelligence for mathematics

"*Even if computer mathematics did a good job of simulating human mathematics, it won't do things the same way humans do.*"

AP, first interview, reflecting on the joint work of Gowers and Ganesalingam (Gowers & Ganesalingam, 2012).

D Selected quotes from interviews with participants in the user evaluation

This appendix presents a selection of illustrative quotes from interviews with participants in the field trial described in Chapter 9. These quotes are referred in brackets in the main text: for instance, [AC3] references the third quote from subject **AC**. Table 20 [§9.1, p. 142] presented a brief description of the interview subjects; the key data is repeated below. Recordings from **AA** and **AB** are missing due to a problem with the recording software; accordingly, data from these interviews is embedded in diagrammatic form in my handwritten notes, and not included below. **BC** dropped out of the study before completing an interview.

AC Selected quotes

German resident, male, mathematics PhD, staff scientist and programmer, long-time PlanetMath user (contacted through PlanetMath).

1. *But these are escape things, is there any documentation. . . because they're not necessarily LaTeX stuffs.*

2. *Yeah, that's great, when you can use everything that's available in LaTeX with PlanetMath, I think that's really positive, because you can't do that with other places, with Wikipedia, you can't build such things.* [. . .] *Because, I'm thinking, if I'm blogging Wordpress, and if I'm doing it, I have to convert everything I've written in LaTeX in Wordpress, and that's a lot of work.*

3. *And, recently you know, I was looking at the activity in PlanetMath, in the past 5 days, since I posted something new, there wasn't much activity in PlanetMath itself.*

4. *But, the funny thing is, I mean, assuming that somebody wrote an article like 10 years ago, what would happen then? He's, like, not anymore active in PlanetMath.*

5. *Another question is – regarding the problems – if I post a problem, can I watch on it? Because I haven't seen this possibility. Let me try again, I cannot remember, but that was one of the issues that was lingering in my mind. Alright, yeah I'm not able to watch* [...] *Oh, no no, I am - "you're not watching this" - so, I'm able to watch, so maybe that's my mistake.*

6. *Well, I was really seriously thinking of putting some really you know... but these are articles, I do not know the validity of them or something... so I can put them in articles... I can put them... I'm not sure if I should put them in problems or so, so I was just a bit confused – I'm not sure where they should land.* [...] *Well, I mean, they are, like you say, open problems... you can put it in problems, right – but suppose you are working on an open problem, and you have certain theories, and you're not really sure if these theories are at all correct. And you just want to put stuffs.*

7. *Every time I have some such issues, I won't hesitate, I'll just bombard you with screen-shots or questions. I know you're quite busy with these things, but it might help. Do you have a bug tracker?* (I tell him that the project is developed on Github, and we use the integrated bug tracker there.) *Oh really! Is it, I mean, do people look at it, I mean, some bug trackers they're just...*

8. *I think what StackOverflow, or, I mean I think it's called MathOverflow, there's also this Math-StackOverflow something, it's fine, but it's not too personalized. With PlanetMath, you can log in, and then you can, maybe you can, it's easier to find, or to categorize or to organize your problems there than in StackOverflow.*

9. *Yeah, I think it's more user friendly when it comes to users having problems with their homework or asking question. For instance, there's this MathOverflow, they filter out, they only entertain people who are a bit of, above graduate level, but in PlanetMath you don't have this problem, I mean this you don't have this kind of...*

10. *Well, you know, for instance I'm doing some private research, mathematical research, purely mathematical, that has no application at all, you know, it just interests me. I was thinking, you know, I could write a blog on it, because I'm not really interested in having publications, because, the problem I see with publications, is that you have to wait for... almost a decade until all the reviewers suddenly say, OK, this is fine and then you can publish it. I mean, I've had experience with it in my Ph. D. years, that you know you'll have to wait a long time because the reviewers don't really have time for it; and then after that, you will have to do the changes that the reviewer asks you, and then after that, it's published. But I would prefer to have it already online,*

I'm really not interested in having my work officially published, because in the end, the people who are getting benefit from are actually the press, and it's not really the scientists who are involved, because in the end, people have to buy my articles, which is not interesting for me. So I was interested in writing blogs about this thing. But now that PlanetMath comes to... you know, I was thinking of, maybe it would be possible to do such, write my researches in PlanetMath.

11. *I think the problem is PlanetMath lacks some of the most trivial things, like, for instance, if I posted something, like about Lie algebra or something, then after one month, I do not see it in my history. Unless it's an article. If I post it in, like as a question, or in a problem set or something... I never see it in my history and if I search for it, it's a catastrophe. You cannot – you're not able to search it.* (I show him the "Score" tab.) *... but this includes also the things that I've viewed.*[*]

BA Selected quotes

UK resident, male, engineering background, enrolled in the Open University's mathematics masters program (contacted through OU mathematics department).

1. *Um, right, I started out with an article* [...] *I did find, however, that when I was typing into it, I was pressing "preview" to see what it looked like, but it would have been really convenient to see what it looked like in some ways, as I went along. But, it wasn't clear – I ended up doing a lot of "save" operations and then carrying on, because I wasn't clear as to whether the thing was going to time out on me, and then all the data would be wiped – because, it's quite a slow process, obviously – if I had left it a few hours, and then went back, and wanted to make some amendments to it, because I couldn't actually see the article, you know, you had to sort of go in, make your amendment, do a preview...*

2. *Um, yeah, because there are so many categories, I found it a bit confusing. Yeah, so, something like "collection", I would think that ought to be something where you would have an editor, who would move articles around, into collections, say, rather than a writer choosing which collection to be in.*

3. *Yes, it's a – it's a different approach really. So, I mean, something – probably something like a collection – it would probably be good if an editor could at least move something, collect articles together, perhaps place some blank articles there – you know, to try to get someone to actually write one.*

[*]More detail was added to user pages after this interview.

4. *Yeah, I mean, that's another thing – you might have a collection, where you have something at, say, at school level, and another one at, I don't know, postgraduate level. And there really isn't going to be any overlap at all.*

5. [E]*veryone's allocated a course tutor, who might take on just a half-dozen students – so, they're not the overall person in charge of the course, by any means.* [...] *It is called Elluminate, spelt with an E, I think it's some Canadian organization; it's got lots of Java in it, but um, it's essentially a whiteboard, with very limited ways in which people can talk, apart from the tutor.* [...] *In OU terms, the forums, essentially, there's a moderator, and they try to dispel any talk about these Tutor Assignments, at all, on these forums.* [...] *Well, you – either* [solve the problems] *yourself – or if you're in difficulty, you should contact your tutor. Well, they're quite helpful, in that they'll give you a hint to set you off in the right direction. And, obviously, if all else fails, these questions are normally in parts, so if you can't do one part, you can just move on to the next. So that is not, if you like, exclude you because you've sort of... failed to get enough marks on your assignment.*

6. *This one, I dropped before the end, and that was Functional Analysis, it was just too, if you like, abstract for me.* [...] *I think that Functional Analysis is just, if you like, it's really something that is so abstract – that, it would be very d... – I think there are some people who can find their way around that, but, for some people, you reach a certain level of abstractness, and they're just lost.* [...] *Well, as it turns out, my current course - does use a few of the ideas of functional analysis, in the sense that you get finite dimensional linear spaces, rather than infinite dimensional spaces... which you can't really visualize, you can't draw pictures, really... so, I mean, I find that quite straightforward; I mean, it's really just ordinary vector spaces, things like that, which is sort of undergraduate stuff.*

BB Selected quotes

UK resident, female, engineering and software background, enrolled in the Open University's mathematics masters program (contacted through OU mathematics department).

1. *Because, I mean, what we've got on PlanetMath is masses of very good math, masses of very good theorems and results and definitions and so on; I just found it a bit difficult to find the one I wanted.*

2. *But is a problem – are you thinking of a problem as something that a teacher has provided for a student to look at, and he has the answer that the student can then go and look at, when he's tackled the problem?* [...] *Because if I was in there thinking*

about answering someone else's question, I want to know, whether that's just a person who wants to know how to do something that I can easily tell them, or whether the tutor has put up something for them to do, and we're not supposed to be answering it, they're supposed to be finding it out for themselves.

3. *I think it would also be nice to have a place to talk with mathematicians, without thinking that you might be making a permanent copy on the website, that ought to be correct and validated and so on, so that you can say, how about this for an answer... where you might be guessing just as much as the other person... but the forums don't seem to be very much used.* [...] *I don't have any problem with them being public, it's that I want it to be quite clear that I'm just speculating, and not giving a definitive proof or theorem or whatever, because most of the site is definitive material.*

4. *So, you would expect if somebody asked a question, that the person answering it wouldn't answer the question, they would just point them in the direction of how to think about it.*

5. *Well, I certainly go into all the OU forums. I certainly ask questions and answer questions. And actually, answering questions is just as much learning as asking them. Because you have to spot how the person who's asking it is looking at something from a funny angle, or upside down, and then explain the way you should be looking at it.* [...] *But don't you just find it interesting, the way other people approach problems? It gives you a whole new view on the world, to see some other, other, um, attitude to the same question that you've just taken for granted – suddenly you get a new perspective.*

6. *I read a section of my maths text, sometimes trying to understand the proofs of the theorems, but more often just reading and trusting that they really work. Then I try to work through some examples or exercises. This often brings up questions about the original material. For example, I ask myself, "Why does the closed contour theorem talk about a function having a primitive while Cauchy's theorem requires an analytic function? What is the significance of this difference?" I think that it is only when I begin to join up bits of the theory in this way that I am really learning.*

7. *I think it's more, the problem is, that when you go out to look at a particular theorem or whatever, you can't very easy get back, you might do your back button, but you might have to do 10, 12 presses to get back to where you started from – whereas if you could have a collection or a topic article or something in your breadcrumbs at the top, then when you'd been wandering around following leads, you could then jump back to where you were.* [...] *I don't think it would need the whole history, I think it would just need the thing that was the topic, whether that's a collection or a topic*

article, a sort of high level thing, that when you went into one, it would remember that until you went into a different one at the high level. It would be a bit difficult to have everything listed where you'd been, all the way, like a history, it would be too complex.

8. *I think they're just a bit difficult to find, which is why I was just suggesting that maybe topics should be brought up to a higher level, and listed separately from articles. So that you can easily find a whole topic, and then like you say that topic gives you the introduction into all the bits that belong to it – probably work better than collections, because being basically an article, you can have all the links, and you can have the words in-between.*

9. *Yeah, also, maybe we could have these sections in smaller steps. You might be able say, I'm looking for a theorem, or I'm looking for a definition, or I'm looking for a method.*

RSP Selected quotes

US resident, male, physics PhD, long-time PlanetMath user and active volunteer (Raymond S. Puzio, requested non-anonymity).

1. *I should mention, with the 3C's and your elaborations – I figured last night that I should explain to you what I meant, because there's been some confusion. So, you are absolutely right about the importance of the code and the organization, however, I guess the way I'm thinking of it is that those two are more, sort of, in the background, they're what's supporting it. Whereas, what I'm thinking is that the content, the catalog, and the community is what the typical user will be interacting with.* [...] *So, that's what I meant, so the idea is, if they're interacting with the 3, hopefully some of them will get interested in looking in the background.*

2. *Also, as I found when I was rummaging through the Library of Congress catalog last week, there's oodles and oodles of elementary math books of problems out there in the public domain – I'm sure we have... we might even have a million problems if we managed to get all of those in.*

3. *Some people might find the shared document and email enough, others might want a voice channel, others might want something else, maybe they want, let's say some algebra program that's running, and a common place to save files.*

4. [A]*s for other uses I'd like, interacting with people on PlanetMath – a lot of that is going to depend on building the user community, for example if I have an interest*

in a research-level topic, there's someone with a similar interest to talk about it with there.

5. *Personally, ok, that information would be good to display, but you also have to involve people, because let's say they posted a lot, but they might be too busy, or might not be in the mood to talk about it – so I would say that improving the interaction along with that would be important.*

6. *So I guess one way of thinking of it is as a replacement of our bio – I mean our user home pages. My idea would be to replace those home pages with some such more robust thing.*

7. *Not just make it easier to respond at your own pace, but also to put that information in a useful form, like here's people's interests, here's the ones who already know you and are contacting you, and ones you might want to contact, etc.*

8. *One thing about content could making it specific. Let's say, we look at* [elided]. *OK, then the community might be, currently the community might be* [**AB**] *and myself and one or two other people. Content, OK, we've got her catalog – her collection – of* [elided]. *Catalog end, I could just extract from the Library of Congress all the books pertinent to that subject. Even if we don't have time to build everything up. So, even if we don't have time to build everything up, at least we could build up that one sub-project really nicely.*

9. *Well, let me tell you* [...] *two specific instances. One thing I did was, we had this project with Matte and Cameron, where we had this project, where the there of us were looking at the counter-examples in topology, and just making sure they were all on PlanetMath. Another one was when – that you probably know – was when Matte tried to do his thing with the real number project, which was basically such a role, the problem was that at that time, it seems he didn't have resources or the support he needed, and he got frustrated.*

10. *So, what I'm thinking is* [...] *I'm not so worried about automating. In the past, I've served that purpose of bridging users together. And I could see myself doing it again, it's just that, for doing that, I should free enough time to make sure I actually have time to interact with people.* (Pause) *So you know, for example, in the past, I've started things like this project we had with topology, and other things, and you know, brought people in. So, one hope I've had is that I'd have an hour or two a day to working on the community side and exactly helping people with such things.*

11. *OK, let's say, Internet Archive, sooner or later, they're going to have them all uploaded as graphics images, if we have the OCR running, then, yeah they could all become*

TEX files, but it might be that only a handful of those ever gets human edited into proper editions, because only they get enough attention.

12. [T]*hings like restarting the community discussions, for example our having our Thursday things once a month, with more people showing up. None of these really require us to build any new software or do anything too time consuming.*

E Illustrative excerpts from field notes

(Day 2 of field work) Did any of your hypotheses get corroborated as a result of your data analysis? If so, which, why and how strongly? I got some good insight into "peer supported problem solving" from the point of view of a mathematics researcher and instructor. With his "instructor hat" on – what he wants is for his 100 students to *talk* with each other. But he wants this because he wants them to turn in better papers. He doesn't care if they work together or whatever! He just wants them to learn the stuff better. He seems, in this regard, to be a really amazing instructor. So, we came up with a model that might work. ("Paper mock up" - I'll have to think more about whether I could implement the thing he needs in software.) With his "research hat" on, we saw that expository text is very different from what might be called "quasi-formal" mathematics – and PlanetMath has a lot of the latter. With things like this, you don't see what the point is. One definition points you to another. I used to write a lot like that. It's interesting to think about how this could be improved (either on PlanetMath or Wikipedia, which, on average, is better in this regard already).

(Day 3 of field work) Did any of your hypotheses get corroborated as a result of your data analysis? If so, which, why and how strongly? "How could PlanetMath help with that?" was an ongoing question. In some cases the answer was very "hypothetical" but in general there tended to be an answer. For instance, looking at some of the rather "bad" (in Jamie's view) articles through the linguistics of mathematics view of Mohan, I can see how PlanetMath might integrate a tool that could *show* how "formal" a given article is, and help lead authors to present their work with suitable expository details. Some of this I could likely do even more cheaply, without waiting for Mohan's work to be completed.

F A critical apparatus, applied to developer discussions

> [I]*t is by headlong flight that things progress and signs proliferate.*
>
> (Deleuze & Guattari, 1980, p. 73)

Table 25 collects the themes from Chapter 9 [§9.2, pp. 143-148], and expands them using key points from the literature review conducted in Chapter 4 (Figure 6 [§4.8, p. 66]). The issues identified in Chapter 7 (summarized as α–ο in Table 16 [§7.5, p. 104]) appear as further annotations. This shows that many of ideas from the initial work with potential stakeholders were still points for discussion in the user evaluation, underscoring their importance, while showing that the underlying issues are not fully resolved.

The mapping here confirms the relevance, for PlanetMath users, of some of the notions coming from the literature. For example, in Chapter 7, it was observed that (λ) "*Users want to be able to make adaptations that leave traces, and simple customizations that don't.*" This is connected to McCalla's idea of "*gradual localized accumulation of information.*" Observing that (ο) "*Computer understanding and formal theories of quality and effective social heuristics are good long-term goals*" connects with McCalla's idea about "*supporting purpose-based use*" illustrates that there is quite a bit of work needed to understand the pragmatic purposes that are involved in mathematical activities.

Chapter 7 gathered ideas about user feedback (β) and involving users in technical work (κ) that had not been encountered in the literature review. There is of course a healthy literature on end-user programming and contributions to free/open source software. This material is relevant to a paragogical approach, but it is easy to see why it did not come up in Chapter 4, given the aims stated there [§4.1, p. 48]. This does suggest the need for caution in applications of the paragogy framework.

To be successful, system developers must find ways to mesh what happens on the site with participants' phenomenal experiences of mathematics.

- human language
- sensory perception
- logic and deduction
- intuition, association, and metaphor
- stimulus/response
- process and time
- socialization [ν]
- content
- community [η]
- catalog [η]
- organization
- software [ι]

There will be a boost in participation and learning-relevant measures if users can express their ideas easily in written language.

Improved graphics and interactive features will also boost participation and learning-relevant measures. [μ]

"Projects" should be modeled as progressive problem solving. [δ]

- personal attention
- public statements
- argumentation and rationale
- shared understanding
- collaborative knowledge
- cultural artifacts
- personal comprehension

The process of progressive, informal, problem solving maps onto the history of interactions and curatorial activities. [γ, ζ]

Roles can regulate behavior in the project, but most roles are ephemeral. Collections based on use tracking can help users create more concrete roles.

- gradual localized accumulation of information [λ]
- focused on the end user
- supporting purpose-based use [ο]

When compared with "closed" production, "open" production comes with a host of new problems to solve.

relevance quality [α] scalability consistency motivation [α]

Users need feedback and follow through: consistency is important in social interactions, not just for expository content. [β, κ]

Concrete applications are an indicator of quality. [ε, ξ]

People will be willing to put in effort if it helps them build "literacies" that they can use to reliably achieve their informal goals.

- getting information
- giving information
- reputation building
- relationship development [θ]
- recreation
- self-discovery
- constructive feedback

Table 25: *A critical apparatus, with mnemonics*

The idea is not that the heuristic concepts in Table 25 should be "*corrected and improved until at last they are worthy of a place in the list of fundamentals*" (Bateson, 1972, p. 5). Rather, much in the same way in which the "grammar" in Table 24 [§10.2, p. 160] could be used to parse mathematical activities, Table 25 can be used as part of a working language that describes the "para-mathematical" activities that are involved in building and improving PlanetMath. Something like this seems necessary if mathematics is to once again be seen as "*a living science*."* In principle, this apparatus can be applied to connect the problems that still exist in the working prototype with the research literature, and the research and development programme. Like the Peeragogy pattern language, this apparatus should be used as a distributed *Roadmap*, or rather, a set of roadsigns. As with the *Roadmap* pattern, it should be regularly revised.

 A similar lexicon should be developed to code the expository features of mathematical text. The "dangerous bend" sign from Bourbaki is a prototype.

Application: coding developer discussions This apparatus is applied here to code eleven outstanding but high-priority issues selected from the Planetary Github tracker. This coding seems particularly suitable for application at the per-ticket level, although it could potentially be used at fine-grained levels to directly annotate text. This helps to connect the theoretical work developed in the thesis with the everyday concerns of developers, and helps to check the completeness of the apparatus.

The dynamics of developer interactions were not studied in detail in the thesis, but the data is online for curious readers, for example: Issue #87 is a prime example of social problem solving. Issue #106 is managed by one person, with updates posted in a blog-like manner. After an initial information-gathering phase, Issue #351 is still open, although preliminary work, on Issue #68, is completed in a branch.† These tickets also help to illustrate my role within the Planetary project, as both "designer" and "software integrator" (I *opened* 332 out of 383, or 87%, of the total issues) as well as programmer (I closed 162, or 78%, of the *closed* issues). The full set of tickets is online,‡ along with additional summary information on developer participation in the Planetary project.§

*Early volumes in Springer's Graduate Texts in Mathematics series include this note in the backmatter: "*A student approaching mathematical research is often discouraged by the sheer volume of the literature and the long history of the subject, even when the actual problems are readily understandable. Graduate Texts in Mathematics, is intended to bridge the gap between passive study and creative understanding; it offers introductions on a suitably advanced level to areas of current research. These introductions are neither complete surveys, nor brief accounts of the latest results only. They are textbooks carefully designed as teaching aids; the purpose of the authors is, in every case, to highlight the characteristic features of the theory. Graduate Texts in Mathematics can serve as the basis for advanced courses. They can either be the main or subsidiary sources for seminars, and they can be used for private study. Their guiding principle is to convince the student that mathematics is a living science.*"

†https://github.com/m-iancu/planetary

‡https://github.com/KWARC/planetary/issues

§https://github.com/KWARC/planetary/contributors

#407 editor needs to be supplemented with a “cheat sheet”

The editor could be made much easier to use, with a cheat sheet and more buttons.

#381 SQLite instead of MySQL

It would be good if contributing developers could get started with an instant deployment, and SQLite would be one way to help with this.

#354 Planetary should do better error reporting for LaTeXML fatal errors (was: silent failure of DruTeXML/LaTeXML for posts containing an unescaped “# symbol”)

It’s very confusing for usrs if they don’t get feedback on errors in source code they submit. Other services, like ShareLaTeX, do a relatively nice job with this, with line-level markup to indicate errors.

#351 killer feature: use git to facilitate instant revisions

With suitable integration, Git could be used within Planetary in order to get the best of both Noösphere-style ownership and wiki-style editing. The work done on the MathHub frontend does not support this sort of feature yet.

#328 Meta-ticket for module cleaning process

To help future developers, the source code needs a lot of refactoring. Drupal has a “submodule” feature that can be used to organize things.

#288 make the user experience faster for contributing authors

Currently, users have to wait for processing which would be better to do in the background.

#88 developer docs

Documentation that will help people who are interested in development work have a good experience may help to bring in more volunteers.

#73 deal with subpath for image conversions (was: send in basepath as LaTeXML flag)

Links/Backlinks between gallery pages and the documents that use the images are not currently available.

#68 SVN/git integration

Even without fancy UI features and awareness of the ownership model, Git could be used directly for storing revisions. This is done in the MathHub branch of Planetary.

#67 build system

🗐, 💾 If LaTeXML changes or if there is some other reason we need to rebuild all of the content, we currently do this with a script, but this does not give feedback on which articles or macros are breaking. There is a separate LaTeXML build system, but this is not integrated with Planetary.

#46 integrate JOBAD into planetary

🔎, 🐞 JOBAD is a Javascript package that has been used on a demonstration basis to build intuitive point-and-click in-document interactions (like per-line corrections and comments).

Abbreviations

AAR After Action Review

APM-Ξ Austin Problems in Mathematics – Cross-Index

CC-By-NC-SA Creative Commons Attribution-NonCommercial-ShareAlike license

CC-By-SA Creative Commons Attribution-ShareAlike license

CMS Content Management System

FLOSS Free/Libre/Open Source Software

GNU GNU's Not Unix

GNU FDL GNU Free Documentation License

HDM Hyperreal Dictionary of Mathematics

KWARC Knowledge Adaptation and Reasoning for Content

LIWC Linguistic Inquiry and Word Count

MOOCs Massive Open Online Courses

MSC Mathematics Subject Classification

P2PU Peer 2 Peer University

RDF Resource Description Framework

RDFa Resource Description Framework-in-attributes

REPL Read-Eval-Print Loop

WYSIWYG What You See Is What You Get

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